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The Highly Regular and Irregular Neutrosophic Graph on Some Special Graphs

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Abstract. This paper is to defined Regular and Irregular Neutrosophic graph on a special graph such as C_4, Bull graph, path on four vertices, Wagner graph, Corona graph, Barbell graph and the highly irregular Neutrosophic graph such as cricket graph and Bull graph. Some more properties and various examples of regular and irregular Neutrosophic graph, highly irregular Neutrosophic graph and Neighbourly irregular Neutrosophic graphs are studied and viewed for totally regular and irregular Neutrosophic graphs. Some properties related to d_2 – regular and irregular graphs are analysed.

Key words: Regular, irregular, Bull graph, cricket graph, Wagner graph, Corona graph, Barbell graph.

1. Introduction:

Some varies properties of fuzzy graphs [FGs] was introduced by Azriel Rosenfold [1]. C. Sekar and N. R. Santhi Maheswari [5] developed and discussed some properties of (2,K) totally regular and irregular fuzzy graphs. Ravi Narayanan, Santhi Maheswari [4] discussed some more properties of highly irregular and totally irregular fuzzy graphs. In this paper, we defines highly irregular and highly regular, and neighbourly irregular Neutrosophic graphs, C_4, Bull graph, cricket graph, path on four vertices, Wagner graph, Corona graph and Barbell graph of Neutrosophic graphs. The condition that provides the connection between these graphs and a characterization of highly, neighbourly Neutrosophic graphs on some special graphs are examined.

The following terminologies Neutrosophic graph , Single valued Neutrosophic Graph and Single valued Neutrosophic Graphs are abbreviated as NG , SVNG and SVNGs.

2. Preliminaries

Let G : (A_N , B_A) be a FG on G* : (N, A) for which the membership function A_N : N → [0,1] and B_A : N x N→ [0,1]. The degree of a node x in G [3] is defined as d (x) =$\sum B_A$(xy) for all xy ∈ A and total degree of a node x in G [3] is td(x) =d(x) + A_N (x) , x ∈

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N . The d_2 - degree of a node a in G [5] is $d_2(a) = \sum B_A^2$ (a,c), where $\sum B_A^2$ (a,c) =Sup {B(a,b) ∧ B(b,c)} . Also B(a,b) = 0 for (a,b)∉A. If every nodes of G is adjacent to the nodes with distinct degrees, then the graph G [4] is said to be highly irregular fuzzy graph and if every nodes of G is adjacent to the nodes with distinct total degrees , then G is said to be highly totally irregular fuzzy graph. A graph G is called neighbourly irregular fuzzy graph [4] if every two adjacent nodes of the graph G has distinct degree and if every two adjacent nodes has distinct total degrees then the graph G [4] is said to be neighbourly totally irregular fuzzy graph.`

3 . The highly Regular and Irregular Neutrosophic Graph :

Definition: 3.1.1

Let G : (A_N , B_A) be a Single valued Neutrosophic Graph (SVNG) on G* : (N, A) is defined as the functions T_A ,T_I,T_F are mapped from N → [0,1] and it is denoted by the degree of truth - membership. indeterminacy - membership and falsity - membership of the element x ∈ N satisfy the following conditions (i) T_B (x,y) ≤ T_A (x) Λ T_A (y) (ii) I_B (x,y) ≥ I_A (x) V I_A (y) and (iii) F_B (x,y) ≥ F_A (x) V F_A (y) ∀ x , $y \in A$, where the functions T_A ,I_A, F_A are mapped from V to [0,1] and it is denoted by the degree of truth membership, indeterminacy membership. falsity - membership of the element x ∈ N such that 0 ≤ T_A (x) +T_I (x) +T_F (x) ≤ 3 ∀ x ∈ N and the functions T_B , I_B , F_B are mapped from A ⊆ N x N → [0,1] and it is denoted by the degree of truth membership, indeterminacy membership of the arc $(x, y) \in A$ such that 0 ≤ T_B (x ,y) + I_B (x,y) + F_B (x,y) ≤ 3 ∀ $(x, y) \in A$ and every node of G adjacent to nodes with distinct degrees. So the graph G is said to be highly irregular Neutrosophic Graph and if every node of G adjacent to nodes with distinct total degrees. So the graph G is said to be highly totally irregular Neutrosophic Graph .

We are giving suitable examples for above definition

Let us consider a SVNGs G on G*. Define A_N (s) =(0.5,0.3,0.2) , A_N (t) = (0.6,0.4,0.3), A_N (u) = (0.7,0.5,0.4) , A_N (v) =(0.8,0.2,0.3) , A_N (w) =(0.9,0.4,0.35) , B_A (s , t) = (0.4,0.5,0.4) , B_A (t , u) =(0.5,0.6,0.5) , B_A (u , v) =(0.4,0.5,0.4) , B_A (v , w) =(0.5,0.6,0.5), B_A (w , s) = (0.4 ,0.5,0.4). Here td(s) = (1.3,1.3,1.0), td (t) =(1.5,1.5,1.2) , td(u) =(1.6,1.6,1.3), td (v) = (1.7,1.3,1.2) , td(w) =(1.8,1.5,1.25).

Theorem: 3.1.2

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a cycle C_4 and the degree of truth – membership, indeterminacy-membership and falsity membership of the arc is n't a constant function, then the graph G is regular SVNG.

Proof:

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Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a cycle C_4 , then the graph G satisfy the following conditions (i) $T_B(x,y) \leq T_A(x) \Lambda T_A(y)$ (ii) $I_B(x,y) \geq I_A(x) \vee I_A(y)$ and (iii) $F_B(x,y) \geq F_A(x) \vee F_A(y)$ $\forall x, y \in A$ $0 \leq T_A(x) + T_I(x) + T_F(x) \leq 3$ $\forall x \in N$ and $0 \leq T_B(x,y) + I_B(x,y) + F_B(x,y) \leq 3$ $\forall (x, y) \in A$. Now, the degree of truth – membership, indeterminacy-membership and falsity membership of the arc is not a constant function, then each node of G has the same degree. So the graph G is regular SVNG.

Example: 3.1.3

Let us consider a SVNGs G on G*. Define A_N (s) =(0.5,0.4,0.3) , A_N (t) = (0.6,0.3,0.2) , A_N (u) = (0.7,0.5,0.4) , A_N (v) =(0.8,0.3,0.4) , B_A (s , t) = (0.4,0.5,0.4) , B_A (t , u) =(0.3,0.6,0.5) , B_A (u , v) =(0.4,0.5,0.4) , B_A (v , s) =(0.3,0.6,0.5). Here d(s) = d(t) = d(u) = d(v) = (0.7,1.1,0.9). So the graph G is regular SVNG.

Theorem: 3.1.4

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a cycle C_4 and the degree of truth – membership ,indeterminacy-membership and falsity membership of the arc is n't a constant function, then the graph G is not totally regular SVNG.

Proof:

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a cycle C_4 , then the graph G satisfy the following conditions (i) $T_B(x,y) \leq T_A(x) \Lambda T_A(y)$ (ii) $I_B(x,y) \geq I_A(x) \vee I_A(y)$ and (iii) $F_B(x,y) \geq F_A(x) \vee F_A(y)$ $\forall x, y \in A$ $0 \leq T_A(x) + T_I(x) + T_F(x) \leq 3$ $\forall x \in N$ and $0 \leq T_B(x,y) + I_B(x,y) + F_B(x,y) \leq 3$ $\forall (x, y) \in A$. Now, the degree of truth – membership, indeterminacy-membership and falsity membership of the arc is not a constant function, then each node of G has the distinct total degree. So the graph G is not totally regular SVNG.

From the above example,

Here td(s) = (1.2,1.5,1.2), td (t) =(1.3,1.4,1.1) , td(u) =(1.4,1.6,1.3), td (v) = (1.5,1.4,1.3). So the graph G is not totally regular SVNG.

Theorem: 3.1.5

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Bull graph and the degree of truth – membership, indeterminacy-membership and falsity membership of the node and arcs are not a constant function, then the graph G is not regular SVNG.

Proof:

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Bull graph, then the graph G satisfy the following conditions (i) $T_B(x,y) \leq T_A(x) \Lambda T_A(y)$ (ii) $I_B(x,y) \geq I_A(x) \vee I_A(y)$ and (iii) $F_B(x,y) \geq F_A(x) \vee F_A(y)$ $\forall x, y \in A$ $0 \leq T_A(x) + T_I(x) + T_F(x) \leq 3$ $\forall x \in N$ and 0

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$\leq T_B(x,y) + I_B(x,y) + F_B(x,y) \leq 3 \ \forall (x,y) \in A$. Now, the degree of truth – membership, indeterminacy-membership and falsity membership of the node and arcs are not a constant function, then each node of G has the distinct degrees. So the graph G is not regular SVNG.

Example: 3.1.6

Let us consider a SVNGs G on G*. Define $A_N(s) =(0.5,0.4,0.3)$, $A_N(t) = (0.6,0.3,0.2)$, $A_N(u) = (0.7,0.5,0.4)$, $A_N(v) =(0.8,0.3,0.4)$, $B_A(s, t) = (0.2,0.5,0.4)$, $B_A(t, u) =(0.3,0.6,0.5)$, $B_A(u, v) =(0.4,0.5,0.4)$, $B_A(v, s) =(0.3,0.6,0.5)$. Here d(s) = (0.5 ,1.1,0.9), d(t) = (0.5,1.1,0.9), d(u) = (0.7 ,1.1,0.9), d(v) = (0.7,1.1,0.9). So the graph G is not regular SVNG.

Remark : 3.1.7

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Bull graph and the degree of truth – membership, indeterminacy-membership and falsity membership of the node and arcs are not a constant function, then the graph G is not totally regular SVNG.

Theorem: 3.1.8

Let G : (A_N , B_A) be a SVNG on G* : (N, A) . Then the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function for all a ∈ N if and only if the following are equivalent.

(i) The graph G is a (k_1, k_2, k_3) regular SVNG

(ii) The graph G is totally $(2, k_1+c_1, k_2+c_2, k_3+c_3)$ regular SVNG.

Proof:

Suppose that the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function. (ie) $A_N(a) = (c_1,c_2,c_3)\ \forall a \in N$.

To prove (i) ⇒(ii). Assume that the graph G is a (k_1, k_2, k_3) regular SVNG. Then $d_2(a) = (k_1, k_2, k_3)\ \forall a \in N$. so $td(a) =(k_1, k_2, k_3) +(c_1,c_2,c_3)=(k_1+c_1, k_2+c_2, k_3+c_3)$. Hence the graph G is totally $(2, k_1+c_1, k_2+c_2, k_3+c_3)$ regular SVNG.

To prove (ii) ⇒ (i) Assume that the graph G is totally $(2, k_1+c_1, k_2+c_2, k_3+c_3)$ regular NG. To prove the graph G is a (k_1, k_2, k_3) regular SVNG. Then $td(a) =(k_1+c_1, k_2+c_2, k_3+c_3) \Rightarrow d(a)=(k_1+c_1, k_2+c_2, k_3+c_3)-(c_1,c_2,c_3)=(k_1, k_2, k_3)\ \forall a \in N$. Hence the graph G is a (k_1, k_2, k_3) regular SVNG.

Conversely, suppose the graph G is a (k_1, k_2, k_3) regular SVNG and totally $(2, k_1 + c_1, k_2+ c_2, k_3+ c_3)$ regular SVNG. To prove : The degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function . Suppose that the degree of truth membership, indeterminacy-membership and falsity membership of the node is not a constant function. Then $A_N(a) \neq A_N(b)\ \forall a, b \in N$. Now $td(a)= td(b) \Rightarrow (k_1, k_2, k_3) + A_N(a)= (k_1, k_2, k_3) +A_N(b)$. It is a contradiction. Hence the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function.

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Theorem: 3.1.9

If a SVN graph G is both (k_1, k_2, k_3) regular NG and totally (k_{11}, k_{22}, k_{33}) regular NG then the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function.

Proof :

Let the graph $G : (A_N, B_A)$ is a (k_1, k_2, k_3) regular NG and totally (k_{11}, k_{22}, k_{33}) regular NG .Then $d(a) = (k_1, k_2, k_3)$ and $td(a) = (k_{11}, k_{22}, k_{33})$ $\forall$ a $\in$ N. $\Rightarrow d(a) + A_N(a) = (k_{11}, k_{22}, k_{33}) \Rightarrow A_N(a) = (k_{11}, k_{22}, k_{33}) - (k_1, k_2, k_3)$. Hence the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function.

Remark : 3.1.10

Let $G : (A_N, B_A)$ be a SVNG on $G^* : (N, A)$. If the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function then the graph G is not both (k_1, k_2, k_3) regular NG and totally (k_1, k_2, k_3) regular NG.

Example: 3.1.11

Let us consider a SVNGs G on G*. Define $A_N(s) = (0.6,0.4,0.5)$, $A_N(t) = (0.6,0.4,0.5)$, $A_N(u) = (0.6,0.4,0.5)$, $A_N(v) = (0.6,0.4,0.5)$, $A_N(w) = (0.6,0.4,0.5)$, $B_A(s, t) = (0.5,0.5,0.6)$, $B_A(t, u) = (0.5,0.5,0.6)$, $B_A(u, v) = (0.4,0.5,0.55)$, $B_A(v, w) = (0.5,0.6,0.7)$, $B_A(w, s) = (0.5,0.6,0.7)$. Here $d(s) = (1.0,1.1,1.3)$, $d(t) = (1.0,1.0,1.2)$, $d(u) = (0.9,1.0,1.15)$, $d(v) = (0.9,1.1,1.25)$, $d(w) = (1.0,1.2,1.4)$, $td(s) = (1.6,1.5,1.8)$, $td(t) = (1.6,1.4,1.7)$, $td(u) = (1.5,1.4,1.65)$, $td(v) = (1.5,1.5,1.75)$, $td(w) = (1.6,1.6,1.9)$. Now, the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function. So the graph G is not both (k_1, k_2, k_3) regular NG and totally (k_1, k_2, k_3) regular NG.

3.2 . The d_2 - Regular and Irregular Neutrosophic Graph :

Theorem: 3.2.1

Let $G : (A_N, B_A)$ be a SVNG such that $G^* : (N, A)$ is a cycle C_4 and the degree of truth – membership, indeterminacy-membership and falsity membership of the arc is n't a constant function, then the graph G is d_2 - regular SVNG.

Proof:

Let $G : (A_N, B_A)$ be a SVNG such that $G^* : (N, A)$ is a cycle C_4 , then the graph G satisfy the following conditions (i) $T_B(x,y) \leq T_A(x) \wedge T_A(y)$ (ii) $I_B(x,y) \geq I_A(x) \vee I_A(y)$ and (iii) $F_B(x,y) \geq F_A(x) \vee F_A(y)$ $\forall x, y \in A$, $0 \leq T_A(x) + T_I(x) + T_F(x) \leq 3$ $\forall x \in N$ and $0 \leq T_B$

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(x ,y) + I_B (x,y) + F_B (x,y) $\leq$ 3 $\forall$ $(x, y) \in A$. Now, the degree of truth – membership, indeterminacy-membership and falsity membership of the arc is not a constant function, then each node of G has the same d_2 - degree. So the graph G is d_2 - regular SVNG .

Example: 3.2 2

Let us consider a SVNGs G on G*. Define A_N (s) =(0.5,0.4,0.3) , A_N (t) = (0.6,0.3,0.2), A_N (u) = (0.7,0.5,0.4) , A_N (v) =(0.8,0.3,0.4) , B_A (s , t) = (0.4,0.5,0.4) , B_A (t , u) =(0.3,0.6,0.5), B_A (u , v) =(0.4,0.5,0.4) , B_A (v , w) =(0.3,0.6,0.5), B_A (w , s) = (0.4 ,0.5,0.4). Here d_2(s) = d_2(t) = d_2(u) = d_2(v) = (0.6,1.0,0.8).

Theorem: 3.2.3

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a cycle C_5 and the degree of truth – membership ,indeterminacy-membership and falsity membership of the arc isn't a constant function, then the graph G is not totally d_2 – regular SVNG.

Proof:

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a cycle C_4 , then the graph G satisfy the following conditions (i) T_B (x,y) $\leq$ T_A (x) Λ T_A (y) (ii) I_B (x,y) $\geq$ I_A (x) V I_A(y) and (iii) F_B (x,y) $\geq$ F_A (x) V F_A(y) $\forall$ $x, y \in A$ $0 \leq T_A$ (x) +T_I (x) +T_F (x) $\leq$ 3 $\forall$ $x \in N$ and $0 \leq T_B$ (x ,y) + I_B (x,y) + F_B (x,y) ≤ 3 $\forall$ $(x, y) \in A$. Now, the degree of truth – membership, indeterminacy-membership and falsity membership of the arc is not a constant function, then each node of G has the distinct total d_2 –degrees. So the graph G is not totally d_2 –regular SVNG.

Example: 3.2.4

Let us consider a SVNGs G on G*. Define A_N (s) =(0.5,0.4,0.45) , A_N (t) = (0.6,0.5,0.53) , A_N (u) = (0.55,0.45,0.45) , A_N (v) =(0.5,0.4,0.45) , A_N (w) =(0.55,0.45,0.44) , B_A (s , t) = (0.3,0.5,0.6) , B_A (t , u) =(0.6,0.5,0.53) , B_A (u , v) =(0.35,0.4,0.52) , B_A (v , w) =(0.4,0.45,0.51), B_A (w , s) = (0.4 ,0.45,0.51). Here td_2(s) = (1.2,1.35,1.49), td_2 (t) =(1.25,1.35,1.56) , td_2(u) =(1.2,1.35,1.49), td_2 (v) = (1.25,1.25,1.48) , td_2(w) =(1.2,1.3,1.46). Here the graph G is not totally d_2 – regular SVNG.

Theorem: 3.2.5

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Bull graph and the degree of truth – membership, indeterminacy-membership and falsity membership of the node and arcs are not a constant function, then the graph G is not d_2 – regular SVNG.

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Proof:

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Bull graph, then the graph G satisfy the following conditions (i) T_B (x,y) ≤ T_A (x) Λ T_A (y) (ii) I_B (x,y) ≥ I_A (x) V I_A (y) and (iii) F_B (x,y) ≥ F_A (x) V F_A (y) ∀ $x , y \in A$ $0 \leq T_A$ (x) +T_I (x) +T_F (x) ≤ 3 ∀ $x \in N$ and 0 ≤ T_B (x ,y) + I_B (x,y) + F_B (x,y) ≤ 3 ∀ $(x, y) \in A$. Now, the degree of truth – membership, indeterminacy-membership and falsity membership of the node and arcs are not a constant function, then each node of G has the distinct d_2 –degrees. So the graph G is not d_2 – regular SVNG.

Remark : 3.2.6

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a cycle with five nodes and the degree of truth – membership, indeterminacy-membership and falsity membership of the node and arcs are not a constant function, then the graph G is not totally d_2 – regular SVNG.

Theorem: 3.2.7

Let G : (A_N , B_A) be a SVNG on G* : (N, A) .Then the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function for all a∈ N if and only if the following are equivalent

(1) The graph G is a d_2 – (k_1 ,k_2 ,k_3) regular NG
(2) The graph G is totally (2,k_1 +c_1,k_2+c_2 ,k_3+c_3) regular NG.
Proof:

Suppose that the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function. (ie) $A_N(a)$ = (c_1,c_2,c_3)∀a ∊ N .
To prove (i) ⇒(ii). Assume that the graph G is a d_2 – (k_1 ,k_2 ,k_3) regular NG. Then$d_2(a)$ = (k_1 ,k_2 ,k_3) ∀a ∊ N. so t$d_2(a)$ =(k_1 ,k_2 ,k_3) +(c_1,c_2,c_3)=(k_1 +c_1,k_2+c_2 ,k_3+c_3). Hence the graph G is totally (2,k_1 +c_1,k_2+c_2 ,k_3+c_3) regular NG
To prove (ii) ⇒ (i) Assume that the graph G is totally (2,k_1 +c_1,k_2+c_2 ,k_3+c_3) regular NG. To prove the graph G is a d_2 – (k_1 ,k_2 ,k_3) regular NG. Then t$d_2(a)$ =(k_1 +c_1,k_2+c_2 ,k_3+c_3) ⇒$d_2(a)$=(k_1 +c_1,k_2+c_2 ,k_3+c_3)-(c_1,c_2,c_3)=(k_1 ,k_2 ,k_3) ∀a ∊ N. Hence the graph G is a d_2 – (k_1 ,k_2 ,k_3) regular NFG. Conversely, suppose the graph G is a d_2 – (k_1 ,k_2 ,k_3) regular NG and totally (2,k_1 + c_1, k_2+ c_2 , k_3+ c_3) regular NG. To prove: The degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function . Suppose that the degree of truth membership, indeterminacy-membership and falsity membership of the node is not a constant function. Then $A_N(a) \neq A_N(b)$ ∀ a ,b ∊ N. Now t $d_2(a)$= t $d_2(b)$ ⇒(k_1 , k_2 , k_3) + $A_N(a)$= (k_1 , k_2 , k_3) +$A_N(b)$. It is a contradiction .Hence the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function.

Theorem: 3.2.8

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If a SVN graph G is both $d_2 -$ (k_1, k_2, k_3) regular NG and totally $d_2 -$ (k_{11}, k_{22}, k_{33}) regular NG then the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function.

Proof : Let the graph G : (A_N , B_A) is a$d_2 -$ (k_1, k_2, k_3) regular NG and totally $d_2 -$ (k_{11}, k_{22}, k_{33}) regular NG .Then $d_2(a) = (k_1, k_2, k_3)$ and $td_2(a) = (k_{11}, k_{22}, k_{33})$ $\forall$ a ∊ N. $\Rightarrow d_2(a) + A_N(a) = (k_{11}, k_{22}, k_{33}) \Rightarrow A_N(a) = (k_{11}, k_{22}, k_{33}) - (k_1, k_2, k_3)$. Hence the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function.

Remark : 3.2.9

Let G : (A_N , B_A) be a SVNG on G* : (N, A) .If the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function then the graph G is not both $d_2 -$ (k_1, k_2, k_3) regular NG and totally $d_2 -$ (k_1, k_2, k_3) regular NG.

Example: 3.2.10

Let us consider a SVNGs G on G*. Define A_N (s) =(0.6,0.4,0.5) , A_N (t) = (0.6,0.4,0.5) , A_N (u) = (0.6,0.4,0.5) , A_N (v) =(0.6,0.4,0.5) , A_N (w) =(0.6,0.4,0.5) , B_A (s , t) = (0.5,0.5,0.6) , B_A (t , u) =(0.5,0.5,0.6) , B_A (u , v) =(0.4,0.5,0.55) , B_A (v , w) =(0.5,0.6,0.7), B_A (w , s) = (0.5 ,0.6,0.7). Here d(s) = (1.0,1.1,1.3), d (t) =(1.0,1.0,1.15) , d (u) =(0.9,1.0,1.15), d (v) = (0.9,1.1,1.25) , d (w) =(0.9,1.0,1.15), td_2(s) = (1.6,1.5,1.8), td_2 (t) =(1.6,1.4,1.65) , td_2(u) =(1.5,1.4,1.65), td_2 (v) = (1.5,1.5,1.75) , td_2(w) =(1.5,1.4,1.65). Now, the degree of truth – membership, indeterminacy-membership and falsity membership of the node is a constant function . But the graph G is not both $d_2 -$ (k_1, k_2, k_3) regular NG and totally $d_2 -$ (k_1, k_2, k_3) regular NG.

3.3. The d_2 –Regular single valued Neutrosophic graph on path of four vertices

Theorem: 3.3.1

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a path on four vertices. If the degree of the arc is (T_B, I_B, F_B) is constant function, then the graph G is $d_2 -$ (k_1, k_2, k_3) regular SVNG .

Proof

Suppose that the degree of the arc $(T_B, I_B, F_B)$is constant function(say)$(T_B, I_B, F_B)(a,b) = (k_1, k_2, k_3)$ $\forall$ (a,b) $\in A$ .Then $d_2(a) = 2(k_1, k_2, k_3)$ $\forall a \in N$. Hence the graph G is $d_2 -$ (k_1, k_2, k_3) regular SVNG .

Remark 3.3.2

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a path on four vertices. If the graph G is $d_2 -$ (k_1, k_2, k_3) regular SVNG, then the degree of the arc (T_B, I_B, F_B) is not a constant function.

Example: 3.3.3

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Let us consider a SVNGs G on G*. Define A_N (a) =(0.6,0.4,0.5) , A_N (b) = (0.7,0.5,0.6) , A_N (c) = (0.8,0.6,0.7) , A_N (d) =(0.9,0.7,0.8) , B_A (a , b) = (0.5,0.6,0.7) , B_A (b , c) =(0.6,0.7,0.8) , B_A (c , d) =(0.5,0.6,0.7) . Here d_2(a) = (0.5,0.6,0.7), d_2 (b) =(0.5,0.6,0.7), d_2(c) =(0.5,0.6,0.7), d_2 (d) = (0.5,0.6,0.7). The graph G is d_2 - (0.5,0.6,0.7) regular SVNG.

Theorem: 3.3.4

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a path on four vertices. If the degree of alternate arcs (T_B,I_B,F_B) have same membership values then the graph G is d_2 – (k_1 ,k_2 ,k_3) regular SVNG where (k_1 ,k_2 ,k_3) ={(k_1 ,k_2 ,k_3) ʌ (k_{11} , k_{22} , k_{33}) }.

Proof

If the degree of alternate arcs (T_B,I_B,F_B) have same membership values, then

B_A (a_i) = { (k_1 ,k_2 ,k_3) if i is odd

(k_{11} , k_{22} , k_{33}) if i is even }

Case (i) If (k_1 , k_2 , k_3) = (k_{11} , k_{22} , k_{33}) then (T_B,I_B,F_B) is constant function .So G is d_2 – (k_1 ,k_2 ,k_3) regular SVNG.

Case (ii) If (k_1 , k_2 , k_3) <(k_{11} , k_{22} , k_{33}) then d_2(a) =(k_1 ,k_2 ,k_3) $\forall\, a \in N$ .So G is d_2 – (k_1 ,k_2 ,k_3) regular SVNG.

Case (iii) If (k_1 , k_2 , k_3) > (k_{11} , k_{22} , k_{33})then d_2(a) =(k_{11} ,k_{22} ,k_{33}) $\forall\, a \in N$ .So G is d_2 – (k_{11} ,k_{22} ,k_{33}) regular SVNG.

Remark : 3.3.5

Let G : (A_N , B_A) be a SVNG such thatG* : (N, A) is a path on four vertices. If the degree of truth , indeterminacy and falsity membership value of the middle arcs is less than the membership value of the remaining arcs, then the graph G is d_2 – (k_1 ,k_2 ,k_3) regular SVNG ,where k = (T_B,I_B,F_B) of the middle arc.

Example: 3.3.6

Let us consider a SVNGs G on G*. Define A_N (a) = (0.8,0.6,0.7), A_N (b) = (0.7,0.5,0.6) , A_N (c) = (0.6,0.4,0.5) , A_N (d) = (0.9,0.7,0.8) , B_A (a , b) = =(0.6,0.65,0.75), B_A (b , c) =(0.5,0.55,0.65) , B_A (c , d) =(0.55,0.71,0.81) B_A (d , a) =(0.5,0.55,0.65) . Here d_2(a) = (1.0,1.10,1.30), d_2 (b) =(1.0,1.10,1.30), d_2(c) =d_2 (d) = (1.0,1.10,1.30)Hence the graph G is d_2 - (1.0,1.10,1.30) regular SVNG

Remark :3.3.7

The above theorem is not totally d_2 – regular SVNG .But the degree of truth , indeterminacy and falsity membership value of the node is not a constant function.

Example: 3.3.8

Let us consider a SVNGs G on G*. Define A_N (a) = (0.6,0.4,0.5), A_N (b) = (0.7,0.5,0.6)

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A_N (c) = (0.8,0.6,0.7) , A_N (d) =(0.9,0.7,0.8) , B_A (a , b) = (0.5,0.75,0.85), B_A (b , c) =(0.6,0.65,0.75) , B_A (c , d) =(0.5,0.75,0.85) . Here d_2(a) = d_2 (b) = d_2(c) = d_2 (d) =(1.0,1.30,1.50). Now td_2(a) = (1.6 , 1.7 ,2.0), td_2(b) = (1.7 , 1.8 ,2.10), td_2(c) = (1.8 , 1.90 ,2.2), td_2(a) = (1.9 , 2.0 ,2.30). Hence the graph G is d_2 - (1.0,1.3, 1.5) regular SVNG. But G is not totally d_2 – regular SVNG .

3.4 .The d_2 – Regular single valued Neutrosophic graph on Wagner graph

Theorem : 3.4.1

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Wagner graph . If the degree of the arc (T_B,I_B,F_B) is constant function, then the graph G is d_2 – (k_1 , k_2 , k_3) regular SVNG .

Proof

Suppose that the degree of the arc (T_B,I_B,F_B) is constant function (ie)(T_B,I_B,F_B) (a,b) = (k_1 ,k_2 ,k_3) ∀ a , b ∈ A. Thend_2 (a) = 5(k_1 ,k_2 ,k_3) ∀ a , b ∈ N. Hence G isd_2 – 5(k_1 ,k_2 ,k_3) regular SVNG .

Theorem : 3.4.2

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Wagner graph . If the graph G is d_2 – (k_1 ,k_2 ,k_3) regular SVNG, then the degree of truth , indeterminacy and falsity membership values of the arcs are not constant function .

Example: 3.4.3

Let us consider a SVNG such that G*: (N,A) is a Wagner graph . Define A_N (a) = (0.3,0.1,0.2), A_N (b) = (0.4,0.2,0.3) , A_N (c) = (0.3,0.1,0.2) , A_N (d) =(0.4,0.2,0.3) , A_N (e) =(0.3,0.1,0.2) A_N (f) =(0.45,0.25,0.35) A_N (g) =(0.35,0.15,0.25) , A_N (h) =(0.45,0.27,0.32) B_A (a , b) = (0.2,0.3,0.4), B_A (b , c) =(0.22,0.35,0.42) , B_A (c , d) =(0.2,0.3,0.4) B_A (d , e) =(0.22,0.35,0.42),B_A (e , f) =(0.2,0.3,0.4),B_A (f , g) =(0.22,0.35,0.42) ,B_A (g , h) =(0.2,0.3,0.4),B_A (h , a) =(0.22,0.35,0.42),B_A (a , d) =(0.2,0.3,0.4),B_A (e , h) =(0.2,0.3,0.4),B_A (f , c) =(0.2,0.3,0.4),B_A (g , b) =(0.2,0.3,0.4). Here the graph G is d_2 - (0.42,0.65,0.82) regular SVNG. But then the degree of truth , indeterminacy and falsity membership value of the arcs are not constant function .

Remark 3.4.4

Let G : (A_N , B_A) be a SVNG such thatG* : (N, A) is a Wagner graph. If the degree of the arc (T_B,I_B,F_B)is constant function, then the graph G is not totally d_2 – (k_1 ,k_2 ,k_3) regular SVNG .

3.5 . The d_2 – regular single valued Neutrosophic graph on Corona graph

Theorem :3.5.1

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Corona graph $C_n \circ K_1$, n> 6 . Then the degree of the arc (T_B,I_B,F_B) is a constant function iff the graph G is not d_2 - regular SVNG .

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Remark :3.5.2

If the degree of the arc $(T_B,I_B,F_B)$is not constant function, then the graph G is not d_2 – regular SVNG .

Theorem :3.5.3

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Corona graph $C_n\circ K_1$, n> 6 . Then the degree of the nodes (T_B,I_B,F_B) are constant function iff the graph G is not totally d_2 - regular SVNG .

Remark :3.5.4

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Corona graph $C_n\circ K_1$, n> 6 . Then the degree ofthe node (T_B,I_B,F_B) is not constant function iff the graph G is not totally d_2 – regular SVNG .

3.6. The d_2 – Regular single valued Neutrosophic graph on Barbell graph

Theorem :3.6.1

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Barbell graph $\mathbf{B_{n,n}}$ of order 2n. If the degree of the arc (T_B,I_B,F_B) is a constant function, then the graph G is d_2 – regular SVNG .

Example: 3.6.2

Let us consider a SVNG such that G*: (N,A) is a Barbell graph . Define A_N (a) = (0.5,0.3,0.4), A_N (b) = (0.6,0.4,0.5) , A_N (c) = (0.7,0.5,0.6) , A_N (d) =(0.4,0.2,0.3) A_N (e) = (0.5,0.1,0.2) , A_N (f) =(0.6,0.21,0.32) , B_A (a , b) = (0.4,0.6,0.7), B_A (b , c) =(0.4,0.6,0.7) , B_A (b , d) =(0.4,0.7,0.8) B_A (d , e) =(0.4,0.6,0.7) , B_A (d, f) =(0.4,0.6,0.7) . Here d_2(a) = (0.8,1.2,1.4), d_2 (b) =(0.8,1.2,1.4), d_2(c)=(0.8,1.2,1.4), d_2 (d) =(0.8,1.2,1.4),d_2(e)=(0.8,1.2,1.4), d_2 (f) =(0.8,1.2,1.4) . Hence the graph G is d_2 - (0.8,1.2,1.4) regular SVNG.

Remark : 3.6.3

The converse part of above theorem need not be true.

Example: 3.6.4

Let us consider a SVNG such that G*: (N,A) is a Barbell graph . Define A_N (a) = (0.5,0.3,0.4), A_N (b) = (0.6,0.4,0.5) , A_N (c) = (0.7,0.5,0.6) , A_N (d) =(0.4,0.2,0.3) A_N (e) = (0.5,0.1,0.2) , A_N (f) =(0.6,0.21,0.32) , B_A (a , b) = (0.4,0.6,0.7), B_A (b , c) =(0.4,0.6,0.7) , B_A (b , d) =(0.45,0.62,0.73) B_A (d , e) =(0.4,0.6,0.7) , B_A (d, f) =(0.4,0.6,0.7) .

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Here $d_2(a) = (0.8,1.2,1.4)$, $d_2(b) = (0.8,1.2,1.4)$, $d_2(c)= (0.8,1.2,1.4)$, $d_2(d) = (0.8,1.2,1.4)$, $d_2(e) = (0.8,1.2,1.4)$, $d_2(f) = (0.8,1.2,1.4)$. The graph G is d_2 - (0.8,1.2,1.4) regular SVNG. But the degree of the arc (T_B,I_B,F_B) is a constant function.

The above theorem is not totally d_2 – regular SVNG. But the degree of truth , indeterminacy and falsity membership values of the nodes are not constant function .

3.7 . The highly Irregular Neutrosophic Graph

Theorem :3.7.1

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Bull graph and the degree of truth – membership, indeterminacy-membership and falsity membership of the arc is n't a constant function, then the graph G is highly irregular SVNFG.

Proof:

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Bull graph , then the graph G satisfy the following conditions (i) $T_B(x,y) \leq T_A(x) \wedge T_A(y)$ (ii) $I_B(x,y) \geq I_A(x) \vee I_A(y)$ and (iii) $F_B(x,y) \geq F_A(x) \vee F_A(y)$ $\forall x, y \in A$ $0 \leq T_A(x) + T_I(x) + T_F(x) \leq 3$ $\forall x \in N$ and $0 \leq T_B(x,y) + I_B(x,y) + F_B(x,y) \leq 3$ $\forall (x,y) \in A$. Now, the degree of truth – membership, indeterminacy-membership and falsity membership of the arc is not a constant function, then every node of G is adjacent to nodes with distinct degree. So the graph G is highly irregular SVNG.

Example: 3.7.2

Let us consider a SVNG such that G*: (N,A) is a Bull graph . Define $A_N(s) =(0.5,0.4,0.3)$, $A_N(t) = (0.6,0.3,0.2)$, $A_N(u) = (0.7,0.5,0.4)$ 0. 5 , $A_N(v) =(0.8,0.3,0.4)$, $A_N(w) =(0.6,0.2,0.3)$ $\mathbf{B_A}(s , t) = (0.2,0.5,0.4)$, $B_A(t , u) =(0.3,0.6,0.5)$, $B_A(u , v) =(0.4,0.5,0.4)$, $B_A(v , w) =(0.1,0.6,0.5)$, Here $d(s) =(0.2,0.5,0.4)$, $d(t) = (0.5,1.1,0.9)$, $d(u) = (0.7,1.1,0.9)$, $d(v) = (0.5,1.1,0.9)$, $d(w) = (0.2,0.6,0.5)$. So the graph G is highly irregular SVNG.

Theorem :3.7.3

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a cricket graph and the degree of truth – membership ,indeterminacy-membership and falsity membership of the arc is n't a constant function, then the graph G is highly totally irregular SVNG.

Proof:

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a cricket graph , then the graph G satisfy the following conditions (i) $T_B(x,y) \leq T_A(x) \wedge T_A(y)$ (ii) $I_B(x,y) \geq I_A(x) \vee I_A(y)$

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and (iii) F_B (x,y) $\geq$ F_A (x) V F_A(y) $\forall$ x , $y \in A$ $0 \leq T_A$ (x) +T_I (x) +T_F (x) $\leq$ 3 $\forall$ x $\in N$ and 0 $\leq$ T_B (x ,y) + I_B (x,y) + F_B (x,y) ≤ 3 $\forall$ $(x, y) \in A$. Now, the degree of truth – membership, indeterminacy-membership and falsity membership of the arc is not a constant function, then every node of G is adjacent to nodes with distinct total degree. So the graph G is highly totally irregular SVNG.

Example: 3.7.4

Let us consider a SVNG such that G*: (N,A) is a Cricket graph .Define A_N (s) =(0.6,0.3,0.1) , A_N (t) = (0.8,0.2,0.3) , A_N (u) = (0.5,0.4,0.3) A_N (v) =(0.6,0.4,0.3) 0.6 , A_N (w) =(0.5,0.4,0.4) , B_A (s , t) = (0.2,0.5,0.6) , B_A (t , u) =(0.3,0.4,0.4) , B_A (u , s) =(0.5,0.9,0.9) , B_A (t , v) =(0.3,0.4,0.3). Here td(s) = (1.3,1.7,1.6), td (t) =(1.6,1.5,1.6) , td(u) =(1.3,1.7,1.6), td (v) = (0.9,0.8,0.6) . So the graph G is highly totally irregular SVNG.

Theorem : 3.7.5

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Bull graph and the degree of truth – membership, indeterminacy-membership and falsity membership of the node and arcs are not a constant function, then the graph G is not neighbourly irregular SVNG.

Proof:

Let G : (A_N , B_A) be a SVNG such that G* : (N, A) is a Bull graph, then the graph G satisfy the following conditions (i) T_B (x,y) $\leq$ T_A (x) Λ T_A (y) (ii) I_B (x,y) $\geq$ I_A (x) V I_A (y) and (iii) F_B (x,y) $\geq$ F_A (x) V F_A(y) $\forall$ x , $y \in A$ $0 \leq T_A$ (x) +T_I (x) +T_F (x) $\leq$ 3 $\forall$ x $\in N$ and 0 $\leq$ T_B (x ,y) + I_B (x,y) + F_B (x,y) ≤ 3 $\forall$ $(x, y) \in A$. Now, the degree of truth – membership, indeterminacy-membership and falsity membership of the node and arcs are not a constant function, then every two adjacent node of G has the same degrees. So the graph G is not neighbourly irregular SVNG.

Example: 3.7.6

Let us consider a SVNGs G on G*. Define A_N (s) =(0.5,0.4,0.3) , A_N (t) = (0.6,0.3,0.2), A_N (u) = (0.7,0.5,0.4) , A_N (v) =(0.8,0.3,0.4) , B_A (s , t) = (0.2,0.5,0.4) , B_A (t , u) =(0.3,0.6,0.5), B_A (u , v) =(0.4,0.5,0.4) , B_A (v , w) =(0.3,0.6,0.5). Here d(s) = (0.2 ,0.5,0.4), d(t) = (0.5,1.1,0.9), d(u) = (0.7 ,1.1,0.9), d(v) = (0.7,1.1,0.9) ,d(w) = (0.3,0.6,0.5). So the graph G is not neighbourly irregular SVNG

Conclusion:

In this paper, a new idea of the highly regular and irregular and highly totally regular and irregular, Neighbourly irregular and Neighbourly totally irregular single valued Netrosophic graphs are introduced. The d_2 – Regular single valued Neutrosophic graph and the d_2 - Regular and Irregular Neutrosophic Graph are studied through some special graph such as Bull graph,

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cricket graph, path on four vertices, Wagner graph, Corona graph and Barbell graph. The single valued Netrosophic graphs, highly irregular and Neighbourly irregular single valued Netrosophic graph are compared through some varies examples. Some properties of regular and irregular single valued Netrosophic graphs are studied through various examples and the results are examined for totally regular and irregular single valued Netrosophic graphs.

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