10.48047/jocaaa.2026.35.06.17

A NOVEL CHARACTERIZATION OF B^*_{CG}- CONTINUOUS FUNCTION VIA CLOSURE GRILL TOPOLOGICAL SPACE

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Abstract

In this paper, we introduce a new structure in topological spaces, referred to as the closure grill, and develop an associated operator. In a grill topological space (X, t, G) , we define a new class of sets, called B^*_{CG} open sets. Using this notion, we introduce the concepts of B^*_{CG} continuous functions and B^*_{CG}-open functions are investigate their fundamental properties. Several examples are provided to illustrate the definitions clearly. We study the properties of B^*_{CG} open sets, including results concerning arbitrary unions and finite intersections, together with appropriate counterexamples. In addition, we introduce the B^*_{CG} closure and B^*_{CG} interior operators also discussed with various type of continuous functions. This paper establishes several characterizations of the proposed concepts and examines their related properties.

Keywords and phrases: Grill, local function, local closure grill function, B^*_{CG} open sets, locally B^*_{CG} continuous function.

AMS Mathematics Subject Classification : 54C05 , 54C10 , 54A05.

I. INTRODUCTION

In 1947, Choquet [3] introduced the concept of grills, which have since been used in various topological investigations to manipulate mathematical concepts, including proximity spaces and their extensions [2,10] and so on. In a previous paper [9], introduced a topology τ_G based on an operator Φ_G, which was developed quite naturally from grill G,equipped with a topological space (X, T). Furthermore, Roy et al. (2007) [8], a detailed description of the topology is provided, along with associated ideas and results. In 2008, B.Roy and M.N.Mukherjee [8,9] conducted similar investigation, introducing a new operator Γ in the context of grill-associated topology, which is a dual of the previously introduced operator Φ .

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In this paper, we define a closure operator $\mho$ using grill and investigate its various fundamental properties. Also, we introduce the new notion of topology using closure grill operator and obtain its characterizations along with several properties

II. PRELIMINARIES

In this research, we adopt certain symbols such as (X, T), $cl(N)$ and $int(N)$ are used to represent that X possesses the topology τ, as well as the closure and interior of N in (X, T), provided N be any arbitrary subset of X. Furthermore, 2^X represents all possible subsets of X. Throughout this work, we will use the notation GT -space to represent a topological space (X, T, G) equipped with G on X.

Definition 2.1 ([3]): A set G consisting of a collection of non-trivial subsets of X, referred as grill equipped with X provided,

a. $\phi \notin G$

b. $N \in G$ with $N \subseteq S \subseteq \mathcal{X} \Rightarrow S \in G$.

c. $N, S \subseteq X$ and $N \cup S \in G \Rightarrow N \in G$ or $S \in G$.

Definition 2.2 In the closure grill topological space $(\mathcal{X}, \mathcal{T}_{CG})$, equipped with grill structure G. We define an operator $\mho : 2^{\mathcal{X}} \to 2^{\mathcal{X}}$, denoted $\mho(S, T)$ for any $S \in 2^{\mathcal{X}}$ (where $2^{\mathcal{X}}$ represents all subsets in $\mathcal{X}$). The mapping is termed the grill closure local function, induced by G and T . The operator $\mho(\mathcal{S})$ (or simply $\mho$) is characterized as follows:

$$\mho(S) = \{ x \in \mathcal{X} : S \cap cl(\mathbf{U}) \in G,\ \forall \mathbf{U} \in T(x) \}.$$

Definition 2.3 [24]: In the (X, T_G) space, equipped with G on X. The mapping $\Psi : 2^x \to 2^x$, defined as $\Psi(S) = S \cup \Phi(S)$, $\forall S \in 2^{\mathcal{X}}$ satisfies Kuratowski closure axioms. This induces T_G, a new unique topology of X referred as $T_G = \{K \subseteq \mathcal{X} : \Psi(\mathcal{X} \setminus K) = \mathcal{X} \setminus K\}$,where $\forall S \subseteq \mathcal{X}$, $\Psi(S) = S \cup \Phi(S) = \mathcal{T}_G - cl(S)$.

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Theorem 2.4 [22]: In the (X,T_G)-space, equipped with G on X . The function Ʊ is characterized as increasing function. If $N \subseteq S \subseteq X$, then $\mho(N) \subseteq \mho(S)$.

Theorem 2.5 [22]: In the (X,T_G)-space. For any two grill G_1 and G_2 on X . If $G_2 \subseteq G_1$, then it follows that $\mho_1(A) \subseteq \mho_2(A)$.

Theorem 2.6: In the space (X,T_G) equipped with G on $\mathcal{X}$ then for any arbitrary subsets $C, S \subseteq \mathcal{X}$

1. $\mho(C \cup S) = \mho(C) \cup \mho(S)$
2. $\mho(\mho(C)) \subseteq \mho(C) = cl(\mho(C)) \subseteq cl(C)$.

Theorem 2.7 In the space (X,T_G) equipped with G on $\mathcal{X}$ then the certain properties are valid for arbitrary subsets C and B of $\mathcal{X}$.

a) $\mho(C) \setminus \mho(S) \subseteq \mho(C \square S)$
b) $\mho(C) \setminus \mho(S) = \mho(C \square B) \setminus \mho(S)$

Theorem 2.8 In the space (X,T_G) equipped G on $\mathcal{X}$ and suppose that $\mathcal{L}, S \subseteq \mathcal{X}$ with $S \notin G$, then $\mho(\mathcal{L} \cup S) = \mho(\mathcal{L}) = \mho(\mathcal{L} \setminus S)$.

Definition 2.9 In the space (X,T_G), for an arbitrary subset $\mathcal{U}$ of X is called

a. ϕ is open if $\mathcal{U} \subseteq \text{int}(\mathcal{U}^*)$
b. G-α-open if $\mathcal{U} \subseteq \text{int}[\Psi(\text{int}\,\mathcal{U})]$
c. G-pre open if $\mathcal{U} \subseteq \text{int}[\Psi(\mathcal{U})]$
d. G-semi open if $\mathcal{U} \subseteq \Psi(\text{int}\,\mathcal{U})$
e. G-β −open if $\mathcal{U} \subseteq cl[\text{int}(\mathcal{U}^*)]$
f. β −pre open if $\mathcal{U} \subseteq cl[\text{int}(\text{cl}(\mathcal{U}))]$

Definition 2.10 A subset I of a X is referred as a $B^*_{\mathfrak{C}G}$ open set in (X, T, G) whenever $\mathsf{I} \subseteq \text{int}(\mho_c(\mathsf{I})) \cap \mho_c(\text{int}(\mathsf{I}))$.

Definition 2.11: A subset I of a space X is called locally $B^*_{\mathfrak{C}G}$-closed if $\mathsf{I} = Z \cap V$, where $Z \in T$ and V closed.

Definition 2.12: A subset A of a topological space (X,T_G) is called $R(C, B^*_{\mathfrak{C}G})$-set if int(A) = $B^*_{\mathfrak{C}G} - \text{int}(A)$.

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III. $B^{*}_{c_{CG}}$ - Continuous Function

The concept of disintegration of continuity on a CGT -space and certain families of sets have been characterized with respect to Grill Topological space. For simplicity, the operator □ $_{CG}$ is denoted by □ $_{C}$ and the closure grill space (X, T, G) is represented as (X, T_{CG}) throughout this paper. Also, a $B^{*}_{c_{CG}}$ continuous function is defined from CGT -space into a topological space.

Definition 3.1. A function k: $(X, T_{CG}) \to (Y, T')$ is $B^{*}_{c_{CG}}$- continuous if $k^{-1}(V)$ is $B^{*}_{c_{CG}}$ open in (X, T_{CG}) for every open set V of (Y, T').

Definition 3.2. A function k: $(X, T_{CG}) \to (Y, T')$ is locally $B^{*}_{c_{CG}}$-closed continuous if $k^{-1}(V)$ is locally $B^{*}_{c_{CG}}$-closed open (X, T_{CG}) for every open set V of (Y, T').

Definition 3.3. A function ω: $(X, T_{CG}) \to (Y, T')$ is $R(C, B^{*}_{c_{CG}})$-continuous if $\omega^{-1}(V)$ is in $R(C, B^{*}_{c_{CG}})$ in (X, T_{CG}) for every open set V of (Y, T').

Example 3.4. Consider $X = \{Eq_a, Eq_b, Eq_c, Eq_d\}$ and $Y = \{Eq_1, Eq_2, Eq_3, Eq_4\}$ with topologies $T = \{\emptyset, \{Eq_a\}, \{Eq_b\}, \{Eq_a, Eq_b\}, \{Eq_a, Eq_b, Eq_d\}, \{Eq_a, Eq_b, Eq_c\}, \{Eq_a, Eq_b, Eq_c, Eq_d\}\}$ and so $T' = \{\emptyset, \{Eq_1\}, \{Eq_2\}, \{Eq_1, Eq_2\}, \{Eq_1, Eq_2, Eq_4\}, \{Eq_1, Eq_2, Eq_3\}, \{Eq_1, Eq_2, Eq_3, Eq_4\}\}$ and $G = \{\{Eq_a\}, \{Eq_d\}, \{Eq_a, Eq_b\}, \{Eq_a, Eq_c\}, \{Eq_a, Eq_d\}, \{Eq_a, Eq_b, Eq_c\}, \{Eq_a, Eq_b, Eq_d\}, \{Eq_a, Eq_c, Eq_d\}, \{Eq_c, Eq_d\}, \{Eq_b, Eq_d\}, \{Eq_b, Eq_c, Eq_d\}, X\}$. The collection of all $B^{*}_{c_{CG}}$-open sets with respect to (X, T_{CG}) is $\{\emptyset, \{Eq_a\}, \{Eq_b\}, \{Eq_a, Eq_b\}, \{Eq_a, Eq_d\}, \{Eq_a, Eq_b, Eq_d\}, \{Eq_a, Eq_b, Eq_c\}, \{Eq_a, Eq_c, Eq_d\}, X\}$. Define a function k : $(X, T_{CG}) \to (Y, T')$ suchaway that $k(Eq_a) = \{Eq_1\}$, $k(Eq_b) = \{Eq_2\}$, $k(Eq_c) = \{Eq_4\}$, $k(Eq_d) = \{Eq_3\}$. This mapping is $B^{*}_{c_{CG}}$- continuous.

Theorem 3.5. Let : $(X, T_{CG}) \to (Y, T')$ be an arbitrary function between CGT-space (X, T_{CG}) and (Y, T'). If X is an extremally disconnected space. Then the subsequent conditions are mutually equivalent.

a. ω is continuous.

b. ω is $B^{*}_{c_{CG}}$- continuous and locally $B^{*}_{c_{CG}}$-closed continuous.

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Proof. Consider an arbitrary function $\omega : (X, T_{CG}) \to (Y, T')$ and X is an extremally disconnected space.

To prove that, (a)⇒ (b): Let X be an extremally disconnected space. Supposet ω is continuous. Let V be openset in Y. Since, ω is continuous so that $\omega^{-1}(V)$ is openset in X and so $\omega^{-1}(V)$ is $B\c{c}_{CG}^{*}$- open and locally $B\c{c}_{CG}^{*}$-closed in X. Consequently, $\omega^{-1}(V)$ is $B\c{c}_{CG}^{*}$- continuous and locally $B\c{c}_{CG}^{*}$-closed continuous.

To prove that, (b)⇒ (a): Assume that ω is $B\c{c}_{CG}^{*}$- continuous and locally $B\c{c}_{CG}^{*}$-closed continuous, then $\omega^{-1}(V)$ is $B\c{c}_{CG}^{*}$- open and locally $B\c{c}_{CG}^{*}$-closed in X and so $\omega^{-1}(V)$ is openset in X ∀ openset V in Y. Consequently, ω is continuous.

Theorem 3.6. Let : $(X, T_{CG}) \to (Y, T')$ be an arbitrary function between TP-space (X, T_{CG}) and (Y, T'). Then the subsequent conditions are equivalent.

a. ω is continuous.

b. ω is $B\c{c}_{CG}^{*}$- continuous and $R(C, B\c{c}_{CG}^{*})$ -closed continuous.

Proof. Consider an arbitrary function : $(X, T_{CG}) \to (Y, T')$ then,

To prove that, (a)⇒ (b): Assume that ω is continuous. Let $\mathcal{G}$ be openset in Y. Since, ω is continuous, then $\omega^{-1}(\mathcal{G})$ is openset in X and so $\omega^{-1}(\mathcal{G})$ is $B\c{c}_{CG}^{*}$- open and $R(C, B\c{c}_{CG}^{*})$ -set in X ∀ openset $\mathcal{G}$ in Y. Consequently, $\omega^{-1}(\mathcal{G})$ is $B\c{c}_{CG}^{*}$- continuous and $R(C, B\c{c}_{CG}^{*})$ continuous.

To prove that, (b) ⇒ (a): Assume that ω is $B\c{c}_{CG}^{*}$- continuous and $R(C, B\c{c}_{CG}^{*})$ continuous, then $\omega^{-1}(\mathcal{G})$ is $B\c{c}_{CG}^{*}$- open and $R(C, B\c{c}_{CG}^{*})$-set in X and so $\omega^{-1}(\mathcal{G})$ is openset in X ∀ openset $\mathcal{G}$ in Y. Consequently, ω is continuous.

Theorem 3.7. Let : $(X, T_{CG}) \to (Y, T')$ is almost $B\c{c}_{CG}^{*}$-openset and weakly $B\c{c}_{CG}^{*}$-continuous function, then ω is almost $B\c{c}_{CG}^{*}$-continuous.

Proof. Let x ∈ X and let $\mathcal{G}$ be an open set in Y containing ω(x). Since, ω is weakly $B\c{c}_{CG}^{*}$-continuous. Therefore, there exists U ∈ $B\c{c}_{CG}^{*}$ (O(x,X))-continuous suchaway that ω(U) ⊂ cl($\mathcal{G}$). Since, ω is almost $B\c{c}_{CG}^{*}$-open, so that ω (U) ⊂ IC(ω (U)) for every $B\c{c}_{CG}^{*}$-open

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set U of x. Consequently, ω (U) ⊂ IC(ω (U)) and $IC(\omega\ (\mathrm{U})) \subset IC(\mathcal{G}\ ) \Rightarrow$ ω (U) $\subset IC(\omega\ (U)) \subset IC(\mathcal{G}\ ) \Rightarrow \omega$ (U) $\subset IC(\mathcal{G}\ )$ Hence, ω is almost $B^*_{\mathfrak{C}_G}$-continuous.

Theorem 3.8. For a function ω: $(X, T_{CG}) \to (Y, T')$, the subsequent conditions are mutually equivalent.

a. ω is almost $B^*_{\mathfrak{C}_G}$ continuous at b ∈ X.

b. x∈int(□ C (ω^{-1} (IC($\mathcal{G}$)))∩ □ C (int(ω^{-1} (IC($\mathcal{G}$)))) ∀ neighbourhood V of ω(b).

c. ω^{-1} ($\mathcal{G}$) ⊂ $B^*_{\mathfrak{C}_G}$−int(ω^{-1} (IC($\mathcal{G}$)) ∀openset $\mathcal{G}$ of Y

Proof. Consider an arbitrary function : $(X, T_{CG}) \to (Y, T')$ then,

To prove (a)⇒ (b): Assume that ω is almost $B^*_{\mathfrak{C}_G}$ continuous at b ∈ X. Let be any neighbourhood of ω(b). Since, ω is almost $B^*_{\mathfrak{C}_G}$ continuous at b, then there exists $U \in B^*_{\mathfrak{C}_G}(O(x, X))$ suchaway that ω(U) ⊂ IC($\mathcal{G}$), then $U \subset \omega^{-1}\left(IC\left(\mathcal{G}\ \right)\right)$. Since, U is $B^*_{\mathfrak{C}_G}$-open, so that b $\in U \subseteq int\left(□_C(U)\right) \cap □_C\left(int(U)\right) \subset int\left(□_C\left(\omega^{-1}\left(IC\left(\mathcal{G}\ \right)\right)\right)\right) \cap$ $□_C\left(int\left(\omega^{-1}\left(IC\left(\mathcal{G}\ \right)\right)\right)\right) \Rightarrow$ b$\in int\left(□_C\left(\omega^{-1}\left(IC\left(\mathcal{G}\ \right)\right)\right)\right) \cap □_C\left(int\left(\omega^{-1}\left(IC\left(\mathcal{G}\ \right)\right)\right)\right)$ ∀ neighbourhood $\mathcal{G}$ of ω(b x)

To prove (b)⇒ (c): Assume that b ∈ int(□ C (ω^{-1} (IC($\mathcal{G}$)))) ∩ □ C (int(ω^{-1} (IC($\mathcal{G}$)))) ∀ neighbourhood $\mathcal{G}$ of ω(b). Let V be an open set in Y containing ω (b), so that ω (b) ∈ $\mathcal{G}$ and b ∈ ω^{-1} ($\mathcal{G}$) and so b ∈ ω^{-1} $(IC(\mathcal{G}\ ))$, then $x \in \omega^{-1}$ $(IC(\mathcal{G}\ ))$ and $x \in int(□_C(\omega^{-1}$ $(IC(\mathcal{G}\ )))) \cap □_C(int(\omega^{-1}$ $(IC(\mathcal{G}\ )))) \Rightarrow x \in \omega^{-1}$ $(IC(\mathcal{G}\ )) \cap$ $int(\mho C(\omega^{-1}$ $(IC(\mathcal{G}\ )))) \cap □_C\left(int\left(\omega^{-1}\left(IC\left(\mathcal{G}\ \right)\right)\right)\right) \Rightarrow$ b $\in B^*_{\mathfrak{C}_G} - int\ \omega^{-1}$ $(IC(\mathcal{G}\ )$.

Also, b ∈ ω^{-1} ($\mathcal{G}$) and $x \in B^*_{\mathfrak{C}_G}$ −int ω^{-1} (IC($\mathcal{G}$) ω^{-1} ($\mathcal{G}$) $B^*_{\mathfrak{C}_G}$− int ω^{-1} (IC($\mathcal{G}$)) ∀ openset $\mathcal{G}$ of Y.

To prove (c)⇒ (a): Assume that, ω^{-1} ($\mathcal{G}$) ⊂ $B^*_{\mathfrak{C}_G}$− int(ω^{-1} (IC($\mathcal{G}$)) ∀ openset $\mathcal{G}$ of Y. Let V be an openset containing H(x), then b ∈ ω^{-1} ($\mathcal{G}$) ⊂ $B^*_{\mathfrak{C}_G} - int(\omega^{-1}$ $(IC(\mathcal{G}\ ))$. Since,

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$U = B\c{c}^*_{CG} - int(\omega^{-1}(IC(\mathcal{G})),$ and so $U \in B\c{c}^*_{CG}(O(\mathrm{b}, X)) \Rightarrow U = B\c{c}^*_{CG}(O(\mathrm{b}, X))$ $\subset \omega^{-1}(IC(\mathcal{G})) \Rightarrow U \subset \omega^{-1}(IC(\mathcal{G})) \Rightarrow \omega(U) \subset (IC(\mathcal{G}))$.

Definition 3.9. A function ω: $(X, T_{CG}) \rightarrow (Y, T_{CG}')$ be the function in closure grill topological spaces, then the mapping ω is called $B\c{c}^*_{CG}$ continuity if the preimage of $B\c{c}^*_{CG}$ openset in Y is $B\c{c}^*_{CG}$ openset in X.

Theorem 3.10. A function ω: $(H, T_{CG}) \rightarrow (Y, T_{CG}')$ is a $B\c{c}^*_{CG}$-continuity iff the preimage of every $B\c{c}^*_{CG}$ closedset in Y is $B\c{c}^*_{CG}$ closedset in H .

Proof. Let ω be a $B\c{c}^*_{CG}$-continuity and let F be a $B\c{c}^*_{CG}$ closedset in Y, then Y\F is $B\c{c}^*_{CG}$ openset in H . Since, ω is $B\c{c}^*_{CG}$-continuity, so that ω^{-1} (Y \ F) is $B\c{c}^*_{CG}$ openset in H . This implies that, H \ ω^{-1}(F) is $B\c{c}^*_{CG}$ openset in H . Therefore, ω^{-1} (F) is $B\c{c}^*_{CG}$ closedset in H . Hence, the preimage of every $B\c{c}^*_{CG}$ closedset in Y is $B\c{c}^*_{CG}$ closedset in H .

On the other hand, assume that the preimage of every $B\c{c}^*_{CG}$ closed in Y is $B\c{c}^*_{CG}$ closed set in H . Suppose K be $B\c{c}^*_{CG}$ openset in Y, then Y \K is $B\c{c}^*_{CG}$ closedset in Y. Consequently, ω^{-1} (Y \K) is $B\c{c}^*_{CG}$ closedset in X, this implies that H \ω^{-1} (K) is $B\c{c}^*_{CG}$ closed in H . Hence, ω^{-1} (K) is $B\c{c}^*_{CG}$ open in H . Therefore, the preimage of all $B\c{c}^*_{CG}$ openset in Y is $B\c{c}^*_{CG}$ openset in H , which shows K is $B\c{c}^*_{CG}$ -continuity

Lemma 3.11. If (X, T_{CG}) and (Y, T_{CG}') are two closure grill topological space, then for any function ω: $(X, T_{CG}) \rightarrow (Y, T_{CG}')$ then subsequent conditions are equivalent.

a. A function ω is a $B\c{c}^*_{CG}$-continuity.

b. Every $B\c{c}^*_{CG}$ closed set in Y has an inverse image that is $B\c{c}^*_{CG}$ closed in X.

c. ω ($B\c{c}^*_{CG}$−cl(Z)) $\subseteq$ $B\c{c}^*_{CG}$−cl(ω (Z)) for every subset Z of X.

d. ω^{-1}($B\c{c}^*_{CG}$−int(Z)) $\subseteq$ $B\c{c}^*_{CG}$−int(ω^{-1} (Z)) ∀ subset Z of Y.

Definition 3.12. A function : $(X, T_{CG}) \rightarrow (Y, T_{CG}')$ is called $B\c{c}^*_{CG}$-openmap if the range of any $B\c{c}^*_{CG}$-openset in X is also $B\c{c}^*_{CG}$-openset in Y.

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Definition 3.13. A function ω: $(X, T_{CG}) \to (Y, T_{CG}')$ is called $B\text{ç}^{*}_{CG}$-closed map if the image of any $B\text{ç}^{*}_{CG}$-closed set in X is also $B\text{ç}^{*}_{CG}$-closed in Y.

Definition 3.14. A function ω: $(X, T_{CG}) \to (Y, T_{CG}')$ is called $B\text{ç}^{*}_{CG}$-homeomorphism if the following conditions are satisfied.

a. ω is one-to-one and onto.

b. ω is $B\text{ç}^{*}_{CG}$ continuous.

c. ω is $B\text{ç}^{*}_{CG}$-open.

Example 3.15. Consider the sets X = $\{Eq_a, Eq_b, Eq_c\}$ and Y = $\{Eq_1, Eq_2, Eq_3\}$ with topologies T = $\{\emptyset, \{Eq_a\}, \{Eq_b, Eq_c\}, X\}$ also T' = $\{\emptyset, \{Eq_1\}, \{Eq_2, Eq_3\}, Y\}$ equipped with $G_1 = \{\{Eq_a\}, \{Eq_b\}, \{Eq_a, Eq_b\}\{Eq_a, Eq_c\}, \{Eq_b, Eq_c\}, X\}$ and $G_2 = \{\{Eq_1\}, \{Eq_3\}, \{Eq_1, Eq_2\}, \{Eq_1, Eq_3\}, \{Eq_2, Eq_3\}, Y\}$. The collection of all $B\text{ç}^{*}_{CG}$ sets with respect to (X, T_{CG}) is $\{\emptyset, \{Eq_a\}, \{Eq_b\}, \{Eq_a, Eq_b\}, \{Eq_b, Eq_c\}, X\}$. The collection of all $B\text{ç}^{*}_{CG}$ sets with respect to (Y, T_{CG}') is $\{\emptyset, \{Eq_1\}, \{Eq_3\}, \{Eq_1, Eq_2\}, \{Eq_2, Eq_3\}, Y\}$. Define a function : $(X, T_{CG}) \to (Y, T_{CG}')$ suchaway that ω $(Eq_a) = \{Eq_1\}$, ω $(Eq_b) = \{Eq_3\}$, ω $(Eq_c) = \{Eq_2\}$. This mapping $B\text{ç}^{*}_{CG}$-is one-to-one, onto and $B\text{ç}^{*}_{CG}$-open. Thus, H is $B\text{ç}^{*}_{CG}$ homeomorphism.

Theorem 3.16. If : $(X, T_{CG}) \to (Y, T_{CG}')$ be a one-to-one and onto mapping, then ω is a $B\text{ç}^{*}_{CG}$-homeomorphism if and only if $B\text{ç}^{*}_{CG}$-closed and $B\text{ç}^{*}_{CG}$-continuous.

Proof. Suppose that ω is a $B\text{ç}^{*}_{CG}$-homeomorphism, then ω is $B\text{ç}^{*}_{CG}$-continuous. Let F be an arbitrary $B\text{ç}^{*}_{CG}$-closed set in (X, T_{CG}), then X\F is (X, T_{CG})-open, ω(X\F) is $B\text{ç}^{*}_{CG}$- open in (Y, T_{CG}'). That is ω(F) is $B\text{ç}^{*}_{CG}$- closed in (Y, T_{CG}'). Thus, the image of every $B\text{ç}^{*}_{CG}$- closed set in X is $B\text{ç}^{*}_{CG}$- closed set in Y. That is, if ω is $B\text{ç}^{*}_{CG}$-closed.

Conversely, Suppose that ω be $B\text{ç}^{*}_{CG}$-closed and $B\text{ç}^{*}_{CG}$-continuous. Let K be $B\text{ç}^{*}_{CG}$-open set in (X, T_{CG}), then X \ K is $B\text{ç}^{*}_{CG}$-closed set in X. Since, ω is $B\text{ç}^{*}_{CG}$ continuous and $B\text{ç}^{*}_{CG}$-closed and so ω (X \K) = Y \ ω (K) is $B\text{ç}^{*}_{CG}$-closed set in Y. Therefore, ω (K) is $B\text{ç}^{*}_{CG}$-closed set in Y. Thus, ω is $B\text{ç}^{*}_{CG}$- open and homeomorphism.

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Definition 3.17. A function ω: $(X, T_{CG}) \to (Y, T_{CG}')$ is said to be somewhat $B\mathfrak{c}^*_{G}$-irresolute if for every $U \in B\mathfrak{c}^*_{G} - O(T_{CG}')$ and $\omega^{-1}(U) \neq \emptyset$, there exists a $B\mathfrak{c}^*_{G}$-open set V in X such that $V \neq \emptyset$ and $V \subseteq \omega^{-1}(U)$

Theorem 3.18. A function : $(X, T_{CG}) \to (Y, T_{CG}')$ and γ : $(X, T'_{CG}) \to (Z, T_{CG}'')$ be any two functions. If H is somewhat $B\mathfrak{c}^*_{G}$-irresolute function and I is $B\mathfrak{c}^*_{G}$ continuity, then I ∘ H is somewhat $B\mathfrak{c}^*_{G}$-irresolute function.

Proof. Let ω: $(X, T_{CG}) \to (Y, T_{CG}')$ be somewhat $B\mathfrak{c}^*_{G}$-irresolute function and γ : $(X, T'_{CG}) \to (Z, T_{CG}'')$ be $B\mathfrak{c}^*_{G}$- continuity. Let $U \in B\mathfrak{c}^*_{G} - O(T_{CG}'')$. Suppose that, $\gamma^{-1}(U) \neq \emptyset$. Since, $U \in B\mathfrak{c}^*_{G} - O\left(T_{CG}''\right)$ and γ is $B\mathfrak{c}^*_{G}$- continuity then, $\gamma^{-1}(U) \in B\,B\mathfrak{c}^*_{G} - O\left(T_{CG}'\right)$. Suppose that, $(\gamma \circ \omega)^{-1}(U) = \omega^{-1}\ \left(\gamma^{-1}(U)\right) \neq \emptyset$. Since, by hypothesis H is somewhat $B\mathfrak{c}^*_{G}$-irresolute function, then there exists a $B\mathfrak{c}^*_{G}$ open set V in X such that $V \neq \emptyset$ and $V \subseteq \omega^{-1}\ \left(\gamma^{-1}(U)\right) \Rightarrow V \subseteq (\gamma \circ \omega)^{-1}(U)$. Consequently, $\gamma \circ \omega$ is somewhat $B\mathfrak{c}^*_{G}$-irresolute function.

Theorem 3.19. Let (X, T_{CG}) and (Y, T_{CG}') be any two closure grill topological space. Suppose $X = A \cup B$ are open subsets of X and ω: $(X, T_{CG}) \to (Y, T_{CG}')$ be a function such that $\omega|A$ and $\omega\,|B$ are somewhat $B\mathfrak{c}^*_{G}$-irresolute function.

Proof. It is incumbent upon us to prove that ω is somewhat $B\mathfrak{c}^*_{G}$-irresolute function. Let $U \in B\mathfrak{c}^*_{G} - O(T')$ and $\omega^{-1}\ (C) \neq \emptyset$. Then,

a. $(\omega|A)^{-1}(U) \neq \emptyset$ (or)
b. $(\omega|B)^{-1}(U) \neq \emptyset$ (or)
c. $(\omega|A)^{-1}(U) \neq \emptyset$ and $(\omega|B)^{-1}(U) \neq \emptyset$

Case: 1 Suppose . $(\omega|A)^{-1}(U) \neq \emptyset$. Since, $\omega|A$ is somewhat $B\mathfrak{c}^*_{G}$-irresolute, ∃ a $B\mathfrak{c}^*_{G}$-openset V in A suchthat $V \neq \emptyset$ and $V \subseteq (\omega|A)^{-1}(U) \subseteq \omega^{-1}(U)$, so that V is $B\mathfrak{c}^*_{G}$-openset X. Therefore, there exists a $B\mathfrak{c}^*_{G}$-openset V in X suchthat $V \neq \emptyset$ and $V \subseteq \omega^{-1}(U)$. Thus, H is somewhat $B\mathfrak{c}^*_{G}$-irresolute function.

Case: 2 The proof for this instance proceeds directly, analogous to the proof provided for Case 1.

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Case: 3 Suppose $(\omega|A)^{-1}(U) \neq \emptyset$ and $(\omega|B)^{-1}(U) \neq \emptyset$from case 1 and case 2, ω is somewhat $B^{*}_{\mathfrak{c}_{CG}}$-irresolute function.

Therefore, the examination of the three aforementioned cases suggests that ω can be characterized as a $B^{*}_{\mathfrak{c}_{CG}}$-irresolute function.

Definition 3.20. A function ω: $(X, T_{CG}) \rightarrow (Y, T_{CG}')$ is said to be somewhat R$-B^{*}_{\mathfrak{c}_{CG}}$ open provided $U \in B^{*}_{\mathfrak{c}_{CG}} - O(T_{CG})$ and $U \neq \emptyset$, $\exists$ a $B^{*}_{\mathfrak{c}_{CG}}$-openset V in Y such that $V \neq \emptyset$ and $V \subseteq H(U)$

Theorem 3.21. If ω: $(X, T_1, G_1) \rightarrow (Y, T_2, G_2)$ be somewhat R $-$ $B^{*}_{\mathfrak{c}_{CG}}$-open an Q be any open subset of X, then $\omega|Q$: $(X, T|Q\ A, G_1) \rightarrow (Y, T_2, G_2)$ is somewhat R$-B^{*}_{\mathfrak{c}_{CG}}$-open function.

Proof. Consider the function : $(X, T_1, G_1) \rightarrow (Y, T_2, G_2)$ be somewhat R $-$ $B^{*}_{\mathfrak{c}_{CG}}$-open function and Q be any open subset of X. Let $H \in B^{*}_{\mathfrak{c}_{CG}} - O(T|Q)$ such that $H \neq \emptyset$. Since, H is $B^{*}_{\mathfrak{c}_{CG}}$-openset in $(X, T_1, G_1) \Rightarrow$ H is $B^{*}_{\mathfrak{c}_{CG}}$-open in $(X, T_1, G_1) \Rightarrow$ H . Then by hypothesis, ω is somewhat R$-B^{*}_{\mathfrak{c}_{CG}}$-open. Therefore, $\exists$ a $B^{*}_{\mathfrak{c}_{CG}}$-openset V in Y, so that $V \subseteq \omega(H)$ and so for any $B^{*}_{\mathfrak{c}_{CG}}$-openset $H \in (Q, T|Q)$ with $H \neq \emptyset$, there exists a $B^{*}_{\mathfrak{c}_{CG}}$-open set V in Y, such away that $V \subseteq \omega(H)$. Hence, H|A is somewhat R$-B^{*}_{\mathfrak{c}_{CG}}$-open function.

CONCLUSIONS

This paper proposes a new type of mathematical transformation, called a $B^{*}_{\mathfrak{c}_{CG}}$-open mapping, along with a broadened category of generalized B∗ CG-mappings. These concepts are developed within the context of extended grill spaces. Our ongoing work will investigate their existence and specific characteristics, particularly when applied to emerging generalized topological structures such as nano and minuscule topological spaces, using innovative contraction methods.

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