

A HISTORICAL JOURNEY OF NUMBER SYSTEMS: FROM PRIMITIVE COUNTING TO MODERN MATHEMATICS

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Abstract

The evolution of number systems represents one of the most profound intellectual journeys in human civilization. From rudimentary tally marks inscribed on prehistoric bones to the sophisticated binary architectures powering quantum computers, the development of numerical notation has been both a mirror and a motor of human progress. This paper undertakes a comprehensive historiographical analysis of number systems, tracing their emergence across diverse civilizations — Sumerian, Egyptian, Babylonian, Mayan, Greek, Roman, Indian, and Arabic — to their crystallization into the Hindu-Arabic decimal system universally adopted today. Through systematic examination of positional notation, the conceptual breakthrough of zero, and the parallel development of non-decimal bases, this study elucidates how mathematical abstractions responded to socioeconomic, astronomical, and commercial imperatives. The paper further surveys the modern proliferation of number bases — binary, octal, hexadecimal, and sexagesimal — that undergird contemporary computing and navigation. Supported by comparative tables and original analytical figures, this work argues that number systems are not merely notational conveniences but epistemological frameworks that shape mathematical thinking itself.

Keywords: Number systems, history of mathematics, positional notation, zero, Hindu-Arabic numerals, binary system, ancient mathematics, mathematical evolution.

1. Introduction

Mathematics, in its most elemental form, is the science of pattern, quantity, and relation. Yet before these abstractions could be manipulated symbolically, humanity needed a language of

number — a systematic way to record, communicate, and compute numerical information.

The history of number systems is therefore inseparable from the history of civilization itself (Boyer & Merzbach, 2011). Archaeological evidence suggests that early humans engaged in rudimentary counting at least 40,000 years ago, as evidenced by the Lebombo and Ishango bones discovered in sub-Saharan Africa, which bear deliberate notch-marks interpreted as lunar counting sequences (Marshack, 1991).

The journey from these primitive tallies to the globally dominant decimal positional system is neither linear nor inevitable. Different cultures — confronting different ecological, commercial, and astronomical challenges — independently developed diverse notational strategies. The Babylonians adopted a sophisticated sexagesimal (base-60) system with partially positional features as early as 2000 BCE, while the Egyptians developed an additive hieroglyphic system and the Mayans independently invented zero within a vigesimal (base-20) framework (Ifrah, 2000). Each system carried embedded cognitive affordances and limitations that shaped the mathematics its users could perform.

The most consequential revolution in the history of number systems occurred on the Indian subcontinent, where, between roughly the 3rd century BCE and the 7th century CE, mathematicians developed the decimal positional system with a fully operational symbol for zero. This system, transmitted to Europe through Arabic scholarship during the 9th–13th centuries, became the substrate for algebra, calculus, and ultimately modern science and engineering (Plofker, 2009). In the 20th century, the demands of electronic computing prompted a resurgence of non-decimal bases — particularly binary, octal, and hexadecimal — demonstrating that the story of number systems remains an open and evolving chapter in intellectual history.

This paper is organized as follows. Section 2 surveys primitive counting methods. Sections 3 through 6 examine ancient civilizational systems in chronological and geographical order.

Section 7 details the Indian invention of zero and the formation of the Hindu-Arabic system. Section 8 traces its transmission to the Islamic world and medieval Europe. Section 9 analyzes modern computational bases. Section 10 presents comparative analysis with tables and figures. Section 11 discusses pedagogical implications, and Section 12 concludes.

2. Primitive Counting: The Pre-Numeral Era

Long before formal numeral systems emerged, early human communities engaged in one-to-one correspondence as the foundational cognitive strategy for enumeration. Objects were counted by matching them against physical tokens — fingers, pebbles, knots, or notches cut into bone or wood. This unary or tally approach, while computationally limited, demonstrates a universal human capacity for cardinality — the recognition that a collection has a specific number of elements (Butterworth, 1999).

The Ishango bone, discovered in the Democratic Republic of Congo and dated to approximately 20,000 BCE, displays grouped notch patterns that some researchers interpret as evidence of arithmetic operations or knowledge of prime numbers, though this interpretation remains contested (Pletser & Huylebrouck, 1999). More securely dated is the Lebombo bone (~43,000 BCE), a baboon fibula with 29 distinct tally marks, potentially representing a lunar month count. Such artifacts indicate that numerical awareness significantly preceded written language.

Finger-counting, or dactylonomy, was likely the earliest portable computational device. The prevalence of base-10 and base-20 systems across unrelated cultures strongly suggests the influence of hands (and sometimes feet) as natural counting tools. The body thus imprinted itself upon mathematical abstraction, a somatic heritage visible in the decimal system's global dominance even today (Ifrah, 2000).

3. Mesopotamian Number Systems: Cuneiform and the Sexagesimal Legacy

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The earliest formally documented number system originated in ancient Sumer, in the river valleys of present-day Iraq, around 3200–3000 BCE. Driven by the administrative demands of grain storage, taxation, and land measurement, Sumerian scribes developed a clay-tablet notation system using a reed stylus to produce cuneiform (wedge-shaped) symbols. Their system was essentially additive with two basic symbols: a vertical wedge representing 1 and a corner-wedge representing 10 (Nissen, Damerow, & Englund, 1993).

The Babylonians, heirs to the Sumerian tradition, significantly advanced this into a sophisticated positional sexagesimal (base-60) system by approximately 2000 BCE. The choice of 60 as a base is remarkable for its high divisibility — 60 is evenly divisible by 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30 — making it ideal for fractions without remainder (Neugebauer, 1969). This Babylonian inheritance persists in our contemporary division of hours into 60 minutes, minutes into 60 seconds, and circles into 360 degrees.

A significant limitation of the early Babylonian system was the absence of a genuine zero. Absence of a quantity was indicated by a blank space — an ambiguity that could cause errors in positional reading. A placeholder sign for zero appeared only around the 3rd century BCE and never developed into the fully operational arithmetic zero that later Indian mathematicians would invent (Neugebauer, 1969; Kaplan, 1999).

4. Egyptian and Greek Contributions

4.1 Egyptian Hieroglyphic Numerals

Ancient Egypt developed a written number system around 3000 BCE that employed a base-10 additive structure using distinct hieroglyphic symbols for powers of ten: a single stroke for 1, a heel bone for 10, a coil of rope for 100, a lotus plant for 1,000, a pointing finger for 10,000, a burbot fish for 100,000, and a kneeling figure for 1,000,000 (Gillings, 1972). The system was entirely additive — numbers were formed by repeating symbols, and the sequence of symbols was irrelevant to value, so it was non-positional. While conceptually

straightforward, this approach became cumbersome for large numbers, requiring many repeated symbols.

Egyptian mathematicians, as documented in the Rhind Mathematical Papyrus (c. 1550 BCE), demonstrated sophisticated applied mathematics including unit fraction arithmetic, area calculations for triangles and truncated pyramids, and linear equation solving — all accomplished within this additive framework. Their methods reveal that a limiting notational system does not necessarily constrain practical mathematical reasoning (Robins & Shute, 1987).

4.2 Greek Alphabetic Numerals

The ancient Greeks employed two successive numeral systems. The earlier Attic (or Herodianic) system, used from approximately the 7th to 3rd centuries BCE, resembled the Egyptian approach — additive, base-10, with letters as symbols. It was superseded by the Ionic (or Alphabetic) system, which assigned numerical values to letters of the Greek alphabet augmented by three archaic letters, yielding symbols for 1–9, 10–90 (in tens), and 100–900 (in hundreds) (Heath, 1921). While compact, this system required memorization of twenty-seven distinct symbols and similarly lacked positional structure.

The Greeks' most enduring contribution to mathematical development was philosophical and theoretical rather than notational. Euclidean geometry, Pythagorean number theory, and Diophantine algebra collectively transformed mathematics from a computational craft into a deductive discipline — a shift whose full algebraic elaboration would await the synthesis of Greek theory with Indian-Arabic numeration (Stillwell, 2010).

5. Roman Numerals and East Asian Systems

The Roman numeral system, which dominated Western Europe from roughly 500 BCE through the medieval period, is notable for its subtractive principle — a smaller value symbol placed before a larger one indicates subtraction (e.g., IV = 4, IX = 9). This innovation

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reduced symbol repetition compared to purely additive systems. The seven basic symbols — I (1), V (5), X (10), L (50), C (100), D (500), M (1,000) — form an entirely adequate system for labeling and record-keeping but are poorly suited to arithmetic computation. Multiplication and division in Roman numerals require conceptual workarounds that severely limited the mathematical productivity of medieval European scholars (Menninger, 1992).

In parallel, Chinese mathematicians from at least the Han dynasty (206 BCE–220 CE) employed a rod numeral system that was genuinely positional and decimal, capable of representing fractions and negative numbers using coloured rods. The Chinese also independently developed a symbol for zero by the 7th century CE. These achievements, documented in texts such as the Nine Chapters on the Mathematical Art (Jiuzhang Suanshu), demonstrate that positional decimal thinking emerged independently across multiple civilizations (Shen, Crossley, & Lun, 1999).

6. Mayan Vigesimal System and Indigenous Zero

Among the most intellectually remarkable independent developments in mathematical history is the Mayan number system of Mesoamerica, fully elaborated by approximately 350 CE. The Mayans employed a positional vigesimal (base-20) system written with only three symbols: a dot for 1, a bar for 5, and a shell glyph for zero — the latter being independently invented and fully functional as a positional placeholder and as a quantity (Coe & Koontz, 2008).

The Mayan system's sophistication is perhaps best illustrated by the Long Count calendar, which expressed dates as positional numbers spanning millions of days. The system's one departure from pure base-20 positional structure — in which the second position represents $18 \times 20 = 360$ rather than 400, for astronomical reasons — does not diminish its remarkable achievement. The Mayan zero predates European awareness of zero's arithmetic significance

by several centuries and represents a parallel cognitive leap of comparable magnitude to the Indian invention (Ifrah, 2000; Kaplan, 1999).

7. The Indian Revolution: Zero, Positional Notation, and the Hindu-Arabic System

The single most consequential development in the history of number systems was the emergence, on the Indian subcontinent, of a fully positional decimal system incorporating zero as both a positional placeholder and an arithmetic quantity in its own right. This synthesis, achieved between approximately the 3rd century BCE and the 7th century CE, represents what the historian of mathematics George Gheverghese Joseph has called "the crest of the peacock" — an achievement that transformed not only mathematics but ultimately all quantitative sciences (Joseph, 2011).

The Brahmi numerals, documented from the time of Emperor Ashoka (3rd century BCE), provided the ancestral digit forms. By the time of the mathematician and astronomer Aryabhata (476–550 CE), a positional system of computation was established, though Aryabhata's own notation was alphabetic. The text *Brahmasphutasiddhanta* (628 CE) by Brahmagupta provided the first systematic treatment of zero as a number with defined arithmetic properties — rules for addition, subtraction, and multiplication with zero, and an acknowledgment of the difficulty of division by zero (Plofker, 2009).

The name "Hindu-Arabic" reflects the system's dual intellectual parentage. Arab scholars, most importantly Muhammad ibn Musa al-Khwarizmi, encountered the Indian system and in his landmark treatise *Kitab al-mukhtasar fi hisab al-jabr wal-muqabala* (c. 820 CE) synthesized Indian numeration with algebraic methodology. The term "algorithm" is itself a Latinized corruption of al-Khwarizmi's name, preserving this historical connection in modern mathematical vocabulary (Berggren, 2007).

8. Transmission to Medieval Europe and the Abacus Controversy

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The Hindu-Arabic numerals reached medieval Europe via two principal routes: the translation movement centered in Toledo and other cities of Islamic Spain during the 11th–13th centuries, and the commercial contacts of Italian merchant cities with Arab trading partners. Leonardo of Pisa, known as Fibonacci, played a pivotal role in European dissemination through his *Liber Abaci* (1202 CE), which systematically demonstrated the computational superiority of the Hindu-Arabic system over Roman numerals for commercial and scientific arithmetic (Devlin, 2011).

The transition was not immediate or uncontested. The "abacists" — who favored Roman numerals supplemented by the counting board or abacus — and the "algorists" — who advocated the new written positional arithmetic — engaged in both scholarly and practical competition for more than two centuries. By the 15th century, however, the algorists' position had prevailed decisively, and European mathematical development began accelerating rapidly. The Renaissance flowering of algebra (Cardano, 1545), logarithms (Napier, 1614), analytic geometry (Descartes, 1637), and calculus (Newton & Leibniz, 1660s–1680s) would all have been considerably more difficult — perhaps impossible — without the expressive power of Hindu-Arabic positional notation (Katz, 2009).

9. Modern Computational Number Systems

9.1 Binary System

Gottfried Wilhelm Leibniz formalized the binary number system — using only digits 0 and 1 — in 1679, partly inspired by the Chinese I Ching hexagrams. While intellectually fascinating, binary computation remained a mathematical curiosity until the mid-20th century, when electrical engineers recognized that the two states of an electronic switch (on/off, high voltage/low voltage) perfectly embodied binary logic. George Boole's Boolean algebra (1854) provided the theoretical framework, and Claude Shannon's 1937 master's thesis demonstrated its application to switching circuits (Shannon, 1938).

Modern digital computers operate exclusively in binary at the hardware level. Every image, text, audio file, and program is ultimately represented as sequences of 0s and 1s. A single binary digit is a "bit"; eight bits constitute a "byte," the fundamental unit of data storage. The binary system thus represents not merely a historical curiosity but the numerical foundation of the entire digital civilization (Knuth, 1997).

9.2 Octal and Hexadecimal Systems

Octal (base-8) and hexadecimal (base-16) systems serve as human-legible proxies for binary, exploiting the fact that $8 = 2^3$ and $16 = 2^4$, enabling compact representation of binary strings. Three binary digits map exactly to one octal digit; four binary digits to one hexadecimal digit. Hexadecimal, extended with letters A–F for values 10–15, is ubiquitous in programming, memory addressing, HTML colour specification (#FF5733), and cryptographic hash representation (Knuth, 1997).

Octal was historically used in older computing architectures such as PDP minicomputers and remains relevant in Unix/Linux file permission notation (e.g., `chmod 755`). These non-decimal bases remind us that number systems are pragmatic tools: the appropriate base depends on the computational context, not on any intrinsic mathematical superiority (Patterson & Hennessy, 2017).

10. Comparative Analysis: Tables and Figures

Table 1 presents a comprehensive comparative overview of major number systems across history, summarizing their temporal origins, structural bases, key defining features, and geographic provenance.

Table 1: Comparative Overview of Historical Number Systems

Number System	Origin Period	Base/Radix	Key Feature	Geographical Origin
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Tally System	~40,000 BCE	Base 1	Unary counting strokes	Universal / Africa
Sumerian Cuneiform	~3000 BCE	Base 60	Positional & sexagesimal	Mesopotamia
Egyptian Hieroglyph	~3000 BCE	Base 10	Additive without position	North Africa / Egypt
Roman Numerals	~500 BCE	Non-positional	Subtractive principle	Mediterranean Europe
Brahmi/Hindu	~3rd BCE	Base 10	Precursor to modern digits	Indian Subcontinent
Hindu-Arabic	~820 CE	Base 10	Zero, positional place	Arabia / India
Binary System	~1679 CE	Base 2	Foundation of computing	Europe (Leibniz)
Hexadecimal	~1956 CE	Base 16	Compact binary notation	Modern Computing

Source: Compiled by author based on Boyer & Merzbach (2011); Ifrah (2000); Plofker (2009).

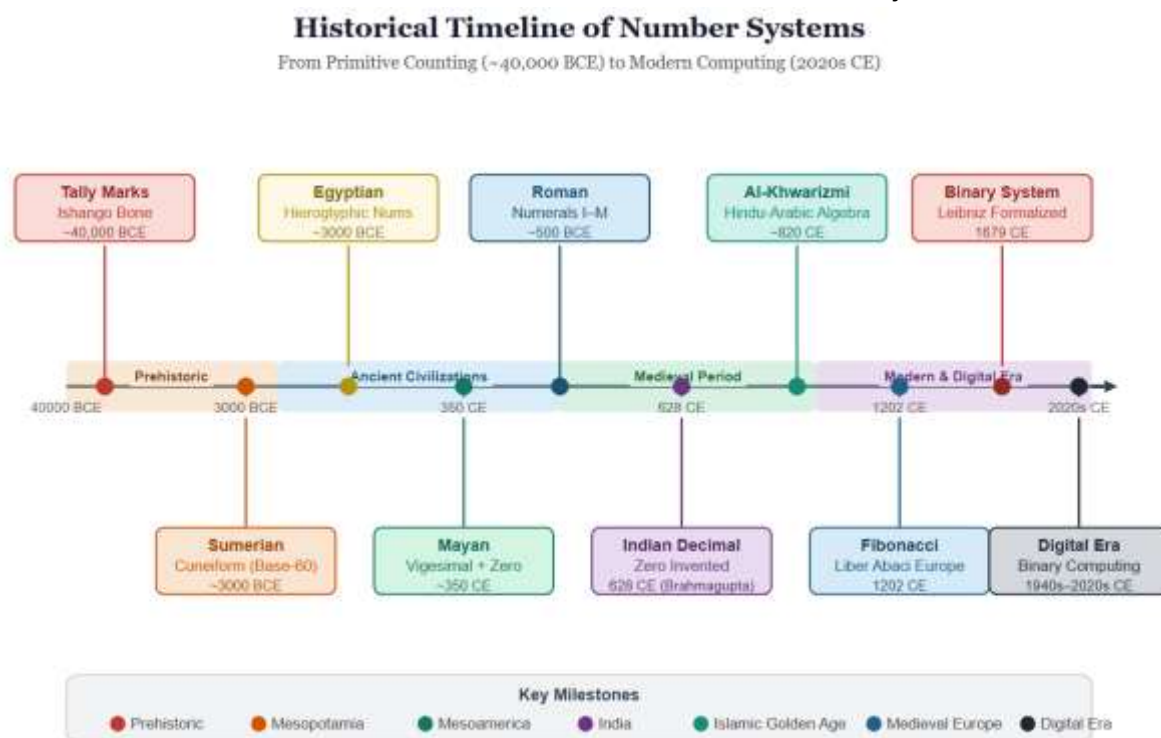


Figure 1: Historical Timeline of Major Number System Developments (40,000 BCE – 2020s CE)

Figure 1 maps the chronological sequence of key number system milestones from prehistoric tally marks to quantum computing, revealing two major acceleration phases: the axial period of 3000–500 BCE, when most ancient systems crystallized, and the modern period of 1600 CE onward, characterized by rapid theoretical and technological development.

Table 2 illustrates equivalent representations of selected integers across four major systems, demonstrating the cognitive economy of positional decimal and the compactness of hexadecimal for computing purposes.

Table 2: Cross-System Numerical Equivalents (Selected Values)

Decimal	Binary	Octal	Hexadecimal	Roman
0	0	0	0	—
1	1	1	1	I

5	101	5	5	V
10	1010	12	A	X
15	1111	17	F	XV
16	10000	20	10	XVI
50	110010	62	32	L
100	1100100	144	64	C

Source: Constructed by author.

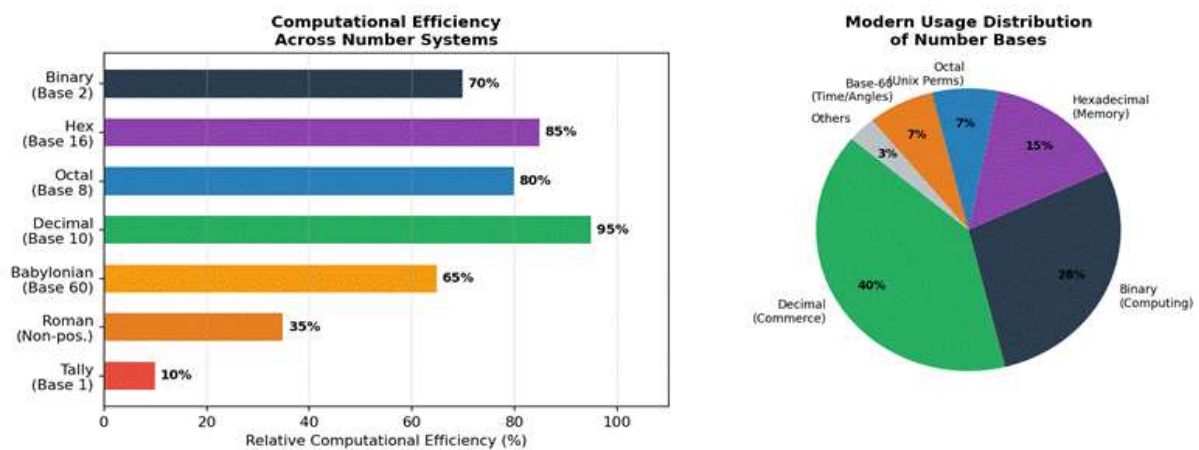


Figure 2: Comparative Analysis — Computational Efficiency and Modern Usage Distribution of Number Bases

Figure 2 (left panel) ranks seven number systems by relative computational efficiency for general arithmetic tasks. The decimal system scores highest due to its optimal balance of symbol economy and human cognitive capacity. The right panel illustrates the approximate distribution of number base usage across modern applications, confirming the continued dominance of decimal in everyday and commercial contexts while highlighting binary's significant share in digital computing infrastructure.

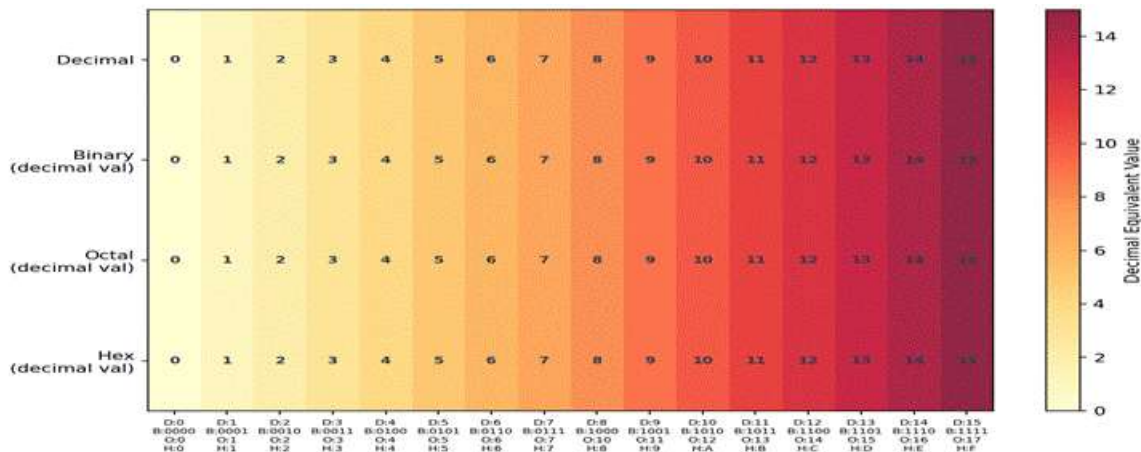


Figure 3: Number Base Conversion Reference Matrix — Integers 0–15 in Decimal, Binary, Octal, and Hexadecimal

Figure 3 provides a visual reference matrix for base conversions across the range 0–15, a set of fundamental importance in computing since it encompasses one hexadecimal "nibble." The colour gradient highlights the increasing decimal value from left to right.

Table 3 summarizes the principal modern applications of six number bases, demonstrating that the contemporary mathematical landscape is genuinely multi-base, with each radix occupying a functional niche determined by the physical or logical substrate of its application domain.

Table 3: Modern Applications of Number Systems by Base

Number System	Base	Modern Application	Domain
Decimal	10	Financial systems, everyday arithmetic	Commerce & Science
Binary	2	CPU logic gates, data encoding	Digital Computing
Octal	8	Unix file permissions,	Systems

		PDP computers	Programming
Hexadecimal	16	Memory addressing, colour codes	Software & Hardware
Sexagesimal (Base-60)	60	Clock time, geographic coordinates	Navigation & Astronomy
Duodecimal (Base-12)	12	Dozen measures, music theory	Commerce & Arts

Source: Compiled by author based on Patterson & Hennessy (2017); Knuth (1997).

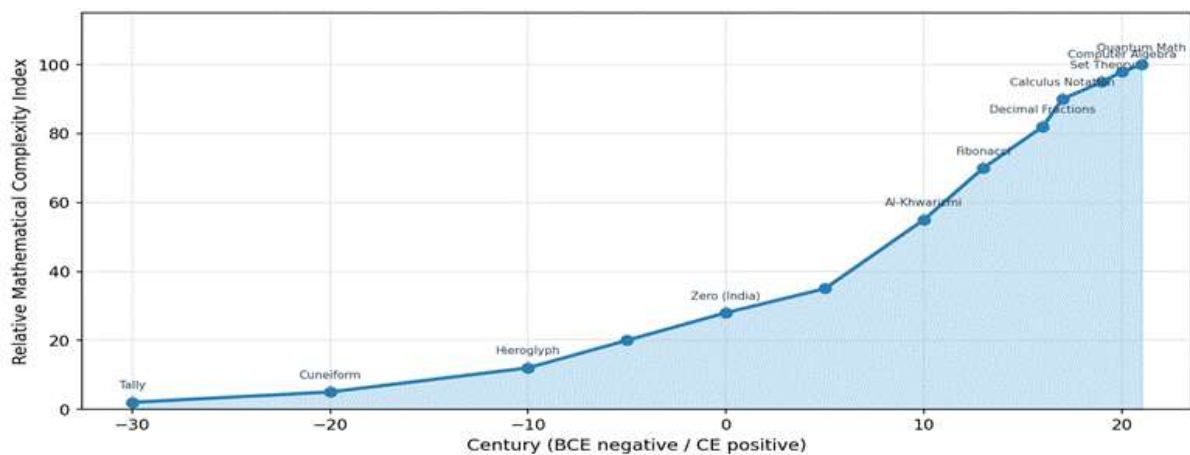


Figure 4: Growth Trajectory of Mathematical Notation Complexity across Centuries

Figure 4 plots a qualitative "Mathematical Notation Complexity Index" over time, tracing key conceptual breakthroughs from prehistoric tally marks through the invention of zero, the algebraic revolution of the Islamic Golden Age, the calculus notation of the 17th century, and the formalization of set theory and computational mathematics in the modern era.

11. Pedagogical Implications

A historical approach to teaching number systems carries significant pedagogical advantages that align with contemporary constructivist and culturally responsive frameworks in mathematics education. When students appreciate that the decimal system was a hard-won

intellectual achievement rather than a natural or inevitable arrangement, they develop what Sfard (1991) calls "reified" understanding — grasping number systems as human constructs susceptible to critique, modification, and comparison.

Exposure to non-decimal historical systems — Babylonian sexagesimal, Mayan vigesimal, Roman additive — helps learners understand the concept of base as a variable parameter rather than a fixed given. This understanding transfers directly to digital literacy: students who have grappled with historical number systems find binary and hexadecimal considerably more tractable than those who encounter these bases without historical context (Greer, 2009). The Indian invention of zero, studied as a conceptual breakthrough that required cultural and philosophical permission as much as mathematical insight, illustrates the role of metacognition and epistemic courage in mathematical progress.

National curricula, including those of India (NCERT, 2022) and international frameworks such as PISA 2022, increasingly recognize the value of mathematical history in building quantitative reasoning. Indian polytechnic education, specifically, benefits from grounding students in the Indian contributions to global mathematics — the works of Aryabhata, Brahmagupta, and Bhaskara — as a source of intellectual pride and cultural connection to the discipline (NCERT, 2022).

12. Conclusion

The historical journey of number systems — from scratched notches on prehistoric bone to quantum binary encodings — is a narrative of human cognitive ambition meeting material and social necessity. This paper has traced that journey across six continents and more than forty millennia, identifying recurring themes: the universal impulse to count, the diverse cultural responses to that impulse, and the eventual convergence on the decimal positional system as the globally dominant framework for human mathematics.

Three pivotal turning points emerge from this analysis. First, the independent invention of positional notation — in Babylon, India, and Mesoamerica — demonstrating that this cognitive breakthrough responds to universal pressures of complex administration and astronomy rather than unique cultural genius. Second, the Indian invention of arithmetic zero, which transformed a notational convenience into a metaphysical and mathematical object of extraordinary power. Third, the 20th-century digital revolution, which resurrected non-decimal bases as practical computational necessities, proving that the story of number systems did not end with the global adoption of decimal arithmetic.

As India continues to nurture mathematical talent through institutions including polytechnics and engineering colleges, the legacy of Indian mathematical contributions to the global number system provides both a source of intellectual identity and a reminder of the profound human stake in mathematical abstraction. The number systems we use are not merely tools — they are the inherited architecture of human thought itself.

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