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Jacobsthal sum cordial labeling for some cycle related graphs

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Abstract

Let G = (V, E) be a graph with p vertices and q edges. An injective function φ:V(G)→{JS_0,JS_1, JS_2,…,JS_n}where JS_n is the n^{th} Jacobsthal sum number,is said to be Jacobsthal sum cordial labeling if the induced function φ^*: E(G)→{0,1}defined by $\varphi^*(uv)$ = [φ(u)+ φ(v)](mod2) satisfies the condition $|e_\varphi(0) - e_\varphi(1)| \leq 1$.

A graph having Jacobsthal sum cordial labeling is called Jacobsthal sum cordial graph. In this paper, we have proved that some of the cycle related graphs such as Dutch windmill graph, Crown graph, Jewell graph, Extended jewell graph, Globe graph, Shadow graph, $P_n \odot C_3$ graph belongs to Jacobsthal sum cordial labeling.

AMS subject classification: 05C78.

Keywords: Labeling, cordial labeling, Jacobsthal sum cordial labeling.

I. Introduction

All graphs, considered here, are simple, undirected ,finite. The idea of labeling techniques was first introduced by Rosa[3] , 1967. In 1987,Cahit[4] introduced the concept of Cordial labeling as a variation of both, graceful and harmonious labeling. Many of the researchers have worked on the various types of cordial labeling. Here we are introducing the concept of Jacobsthal sum cordial labeling. In this paper,we prove that some of the cycle related graphs are Jacobsthal sum cordial labeling.

2. Basic Terminology and Results on Jacobsthal sum cordial labeling

Definition:2.1

Dutch windmill graph D_m^n is obtained by taking n copies of the cycle C_m with vertex is common.

Definition:2.2

The Jewell graph J_n is obtained from a 4- cycle with vertices x,y,u,v by joining x and y with a prime edge and also by appending an edge from u and v which meets at common vertices v_i , 1≤i≤n.The prime edge in a jewell graph is defined to be the edge joining the vertices x and y.

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Definition:2.3

The Extended jewell graph EJ_n is obtained from jewell graph with out prime edge J_n*by appending arbitrary vertices in J_n*such that a way that they all are connected to vertex x and vertex y.

Definition:2.4

The Jewell graph without prime edge J_n* is obtained by removing prime edge in jewell graph J_n.

Definition:2.5

A Globe graph Gl_n is a graph obtained from two isolated vertex are joined by n paths of length two.

Definition:2.6

The shadow graph $D_2(G)$, is constructed by taking two copies of G, namely G itself and G ', and by joining each vertex u in G to the neighbors of the corresponding vertex u ' in G '

Definition:2.7

i). The Jacobsthal sequence is an additive sequence defined by $J_n= J_{n-1}+2J_{n-2}$ with initial terms $J_0 = 0$ and $J_1=1$ and the numbers in the sequence are called Jacobsthal numbers. The Jacobsthal numbers are 0,1,1,3,5,11,21,43,85,….

ii). The Jacobsthal sum numbers is also an additive sequence of Jacobsthal numbers defined by $JS_n= JS_{n-1}+J_n$, n≥ 1. with initial terms be $JS_0 = 0,J_1=1$ and the Jacobsthal sum numbers are 1,2,5,10,21,42,85,170,341,682,1365,…

Definition 2.8

An injective function φ:V(G)→{JS_0, JS_1, JS_2,…,JS_n}where JS_n is the n^{th} Jacobsthal sum numbers, is said to be Jacobsthal sum cordial labeling if the induced function

φ^*: E(G) →{0,1} defined by $\varphi^*(uv)$ = [φ(u)+ φ(v)](mod2) satisfies the condition $|e_\varphi(0) - e_\varphi(1)| \le 1$.

A graph having Jacobsthal sum cordial labeling is called Jacobsthal sum cordial graph.

Theorem 2.1

The Dutch windmill graph D_4^n is Jacobsthal sum cordial labeling, for all n ≥ 2.

Proof:

Let D_4^n be the Dutch windmill graph. Let {x, x_i/1≤i≤m, y_i/1≤i≤n} be the vertices of the D_4^n and {xx_i/1≤i≤m, x_1y_1, x_2y_1, x_3y_2, x_4y_2} be the edges of D_4^n,

Clearly, $|V(D_4^n)| = 3n+1$, $|E(D_4^n)| = 4n$

Define φ : $V(D_4^n)$→{JS_0, JS_1, JS_2,…,JS_{3n}} by

$\varphi(x) = JS_0$, $\varphi(x_i) = JS_i$, i = 1,2,3,…,2n

$\varphi(y_i) = JS_{2n+i}$, i= 1,2,...,n

Hence, $e_\varphi(0)$ = 2n , $e_\varphi(1)$ = 2n

From the above labeling pattern, $|e_\varphi(0) - e_\varphi(1)| \le 1$. Hence the Dutch windmill graph D_4^n is Jacobsthal sum cordial labeling for all n ≥ 2.

Example 2.1 : Jacobsthal sum cordial of Dutch windmill graph D_4^3

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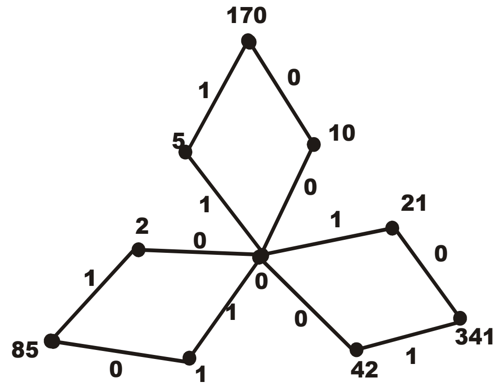

Figure2.1

Theorem 2.2

The crown graph CW_n is Jacobsthal sum cordial labeling, for all $n \geq 3$.

Proof:

Let CW_n be the crown graph. Let $\{x_i/1\leq i\leq n\}$ be the vertices of the cycle graph and $\{y_i/1\leq i\leq n\}$ be the vertices joined to $\{x_i/1\leq i\leq n\}$ and $\{x_ix_{i+1}/1\leq i\leq n-1, x_iy_i/1\leq i\leq n\}$ be the edges of CW_n with $|V(CW_n)| = 2n$, $|E(CW_n)| = 2n$.

Define $\varphi : V(CW_n)\rightarrow\{JS_0, JS_1, JS_2,\ldots,JS_{2n-1}\}$ by

$\varphi(x_1) = JS_0$, $\varphi(x_2) = JS_1$, $\varphi(x_3) = JS_3$, $\varphi(x_4) = JS_2$, $\varphi(x_5) = JS_4$, $\varphi(x_6) = JS_5$, $\varphi(x_7) = JS_7$, $\varphi(x_8) = JS_6$, $\varphi(x_9) = JS_8$, $\varphi(x_{10}) = JS_9$, Continuing like this, we get $\varphi(x_n) = JS_n$, or JS_{n-1}.

$\varphi(y_i) = JS_{n-1+i}$, $i = 1,2,\ldots,n$. Hence $e_\varphi(0) = n$ $e_\varphi(1) = n$. From the above labeling pattern, $|e_\varphi(0) - e_\varphi(1)| \leq 1$.Thus the crown graph CW_n is Jacobsthal sum cordial labeling, for all $n \geq 3$.

Example 2.2 : Jacobsthal sum cordial of the crown graph CW_6

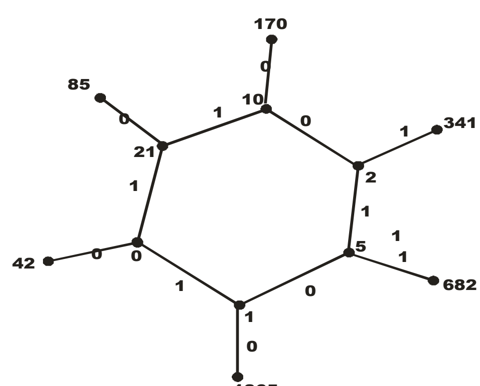

Figure 2.2

Theorem 2.3

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The Jewell graph J_n is Jacobsthal sum cordial labeling, for all $n \geq 1$.

Proof:

Let J_n be the Jewell graph. Let p, q, r, s be the 4- cycle vertices and by joining r and s with a prime edge and also by appending an edge from p and q which meets at common vertices $\{x_i / 1 \leq i \leq n\}$.

Now, $V(J_n) = \{p, q, r, s$ and $xi/1 \leq i \leq n\}$ and $E(J_n) = \{pr, ps, qr, qs, rs, px_i, qx_i / i=1,2,\ldots,n\}$. Clearly, $|V(J_n)| = n+4$, $|E(J_n)| = 2n+5$

Define $\varphi : V(J_n) \rightarrow \{JS_0, JS_1, JS_2, \ldots, JS_{n+3}\}$ by

$\varphi(p) = JS_0$, $\varphi(q) = JS_3$,

$\varphi(r) = JS_2$, $\varphi(s) = JS_1$

$\varphi(x_i) = JS_{i+3}$, $i = 1, \ldots, n$

Hence $e_\varphi(0) = n + 2, e_\varphi(1) = n + 3,$ From the above labeling pattern, $|e_\varphi(0) - e_\varphi(1)| \leq 1$.

Thus the Jewell graph J_n is Jacobsthal sum cordial labeling, for all $n \geq 1$.

Example 2.3 : Jacobsthal sum cordial of the jewell graph J_2

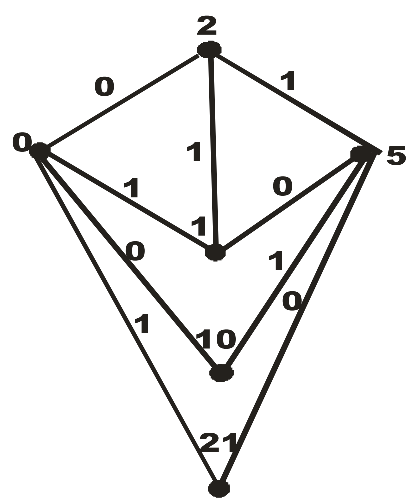

Figure 2.3

Theorem 2.4

The extended Jewell graph EJ_n is Jacobsthal sum cordial labeling, $n \geq 1$.

Proof:

Let EJ_n be the extended Jewell graph. Let p, q, r, s be the 4- cycle vertices and by joining arbitrary vertices $\{y_i / \leq i \leq n\}$ in J_n*such a way that they all are connected to vertex r and s and by appending an edge from p and q which meets at common vertices x_i, $1 \leq i \leq n$.

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Now, $V(EJ_n) = \{p, q, r, s, x_i/1\leq i\leq n, y_j/1\leq j\leq m\}$ and $E(EJ_n) = \{pr, rq, qs, ps, ry_j, sy_j/1\leq j\leq m, px_i, qx_i / 1\leq i\leq n\}$.

Clearly,

$|V(EJ_n)| = m+n+4$, $|E(EJ_n)| = 2n+2m+4$

Define $\varphi : V(EJ_n)\rightarrow\{JS_0, JS_1, JS_2,\ldots,JS_{m+n+4}\}$ by

$\varphi(p) = JS_0$, $\varphi(q) = JS_3$,

$\varphi(r) = JS_2$, $\varphi(s) = JS_1$

$\varphi(x_i) = JS_{i+3}$, $i = 1,\ldots,n$ $\varphi(y_j) = JS_{n+3+j}$, $j = 1,\ldots,m$

Hence, $e_\varphi(0) = m + n + 2, e_\varphi(1) = m + n + 2,$ From the above labeling pattern, $|e_\varphi(0) - e_\varphi(1)| \leq 1$.Thus the extended Jewell graph EJ_n is Jacobsthal sum cordial labeling, for all $n \geq 1$.

Example 2.4 : Jacobsthal sum cordial of the extended jewell graph EJ_3

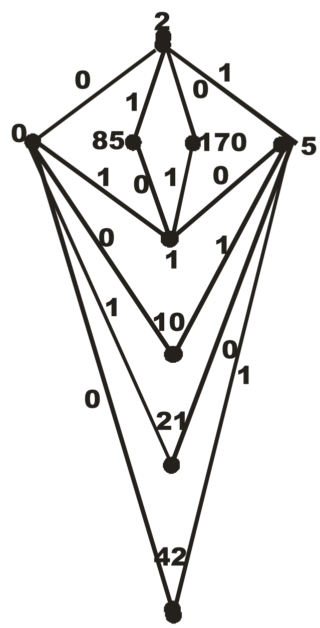

Figure 2.4

Theorem 2.5

The Jewell graph without prime edge J_n^* is Jacobsthal sum cordial labeling, $n \geq 1$.

Proof:

Let J_n^* be the Jewell graph without prime edge. Let p, q, r, s be vertices of 4 - cycle graph and $\{x_i/1\leq i\leq n\}$ be the common vertices joined to p and q. Hence $V(J_n^*) = \{p, q, r, s, x_i/1\leq i\leq n\}$ and $E(J_n^*) = \{pr, rq, ps, sq, px_i , qx_i /1\leq i\leq n\}$ .Clearly,

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$|V(J_n^*)| = n+4$, $|E(J_n^*)| = 2n+4$

Define $\varphi : V(J_n^*) \rightarrow \{JS_0, JS_1, JS_2, \ldots, JS_{n+3}\}$ by

$\varphi(p) = JS_0$, $\varphi(q) = JS_3$,

$\varphi(r) = JS_2$, $\varphi(s) = JS_1$

$\varphi(x_i) = JS_{i+3}$, $i = 1, \ldots, n$

Hence, $e_\varphi(0) = n + 2, e_\varphi(1) = n + 2,$ From the above labeling pattern, $|e_\varphi(0) - e_\varphi(1)| \leq 1$.Thus the Jewell graph without prime edge J_n^* is Jacobsthal sum cordial labeling, $n \geq 1$.

Example 2.5 : Jacobsthal sum cordial of the jewell graph without prime edge J_2^*

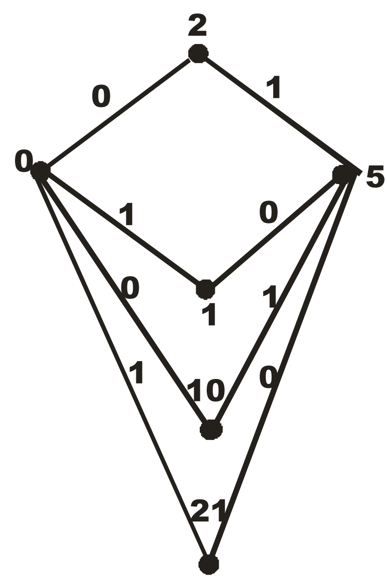

Figure 2.5

Theorem2.6

The globe graph Gl_n is Jacobsthal sum cordial labeling, for all $n \geq 2$.

Proof:

Let Gl_n be the globe graph. Let $\{p, q, x_i / 1 \leq i \leq n\}$ be the vertices of the Gl_n and $\{px_i, qx_i / 1 \leq i \leq n\}$are the edges of Gl_n .

Clearly, $|V(Gl_n)| = n+2$, $|E(Gl_n)| = 2n$

Define $\varphi : V(Gl_n) \rightarrow \{JS_0, JS_1, JS_2, \ldots, JS_{n+1}\}$ by

$\varphi(p) = JS_0$, $\varphi(q) = JS_1$,

$\varphi(x_i) = JS_{i+1}$, , $i = 1, \ldots, n$

Hence, $e_\varphi(0) = n, e_\varphi(1) = n,$ From the above labeling pattern, $|e_\varphi(0) - e_\varphi(1)| \leq 1$.Thus the globe graph Gl_n is Jacobsthal sum cordial labeling, for all $n \geq 2$.

Example 2.6 : Jacobsthal sum cordial of the globe graph Gl(5)

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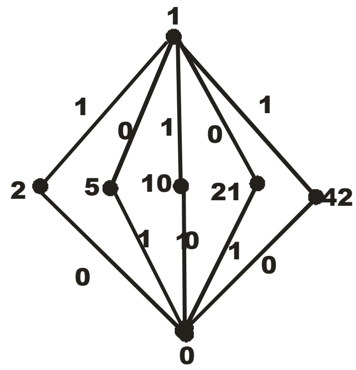

Figure 2.6

Theorem2.7

The Shadow graph $D_2(P_n)$ is Jacobsthal sum cordial labeling, for all $n \geq 2$.

Proof:

Let $D_2(P_n)$ be the shadow graph. Let $\{ x_i, x_i' / 1 \leq i \leq n\}$ be the vertices of $D_2(P_n)$ and $\{x_ix_{i+1}, x_i'x_{i+1}', x_i'x_{i+1}, x_ix_{i+1}' / 1 \leq i \leq n-1\}$ are the edges of $D_2(P_n)$.

Clearly, $|V(D_2(P_n))| = 2n$, $|E(D_2(P_n))| = 4n-4$

Define $\varphi : V(D_2(P_n)) \rightarrow \{JS_0, JS_1, JS_2, \ldots, JS_{2n-1}\}$ by

$\varphi(x_1) = JS_1$,

$\varphi(x_{1+i}) = JS_{1+2i}$, i=1,2,…,n-1

$\varphi(x_1') = 0$

$\varphi(x_{1+i}') = JS_{2i}$, i=1,2,…,n-1

Hence, $e_\varphi(0) = 2(n-1), e_\varphi(1) = 2(n-1)$, From the above labeling pattern,

$|e_\varphi(0) - e_\varphi(1)| \leq 1$.Thus the Shadow graph $D_2(P_n)$ is Jacobsthal sum cordial labeling, for all $n \geq$

2.

Example 2.7 : Jacobsthal sum cordial of the Shadow graph D_2 (P_3)

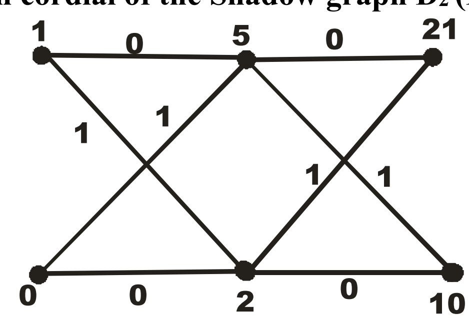

Figure 2.7

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Theorem2.8

The graph $P_n \odot C_3$ is Jacobsthal sum cordial labeling, for all $n \geq 2$.

Proof:

Let $P_n \odot C_3$. Let $\{x_i, x_i' / 1 \leq i \leq n\}$ be the vertices of $P_n \odot C_3$ and $\{x_i x_{i+1} / 1 \leq i \leq n-1, x_1' x_2', x_3' x_4', x_5' x_6', \ldots, x_{2n-1}' x_{2n}'\}, \{x_1 x_1', x_1 x_2', x_1' x_2', x_2 x_3', x_2 x_4', x_3' x_4', \ldots, x_n x_{2n}', x_n x_{2n-1}', x_{2n}' x_{2n-1}'\}$are the edges of $P_n \odot C_3$.

Clearly, $|V(P_n \odot C_3)| = 3n$, $|E(P_n \odot C_3)| = 4n-1$

Define $\varphi : V(D_2(P_n)) \rightarrow \{JS_0, JS_1, JS_2, \ldots, JS_{3n-1}\}$ by

$\varphi(x_1) = JS_0$,

$\varphi(x_{1+i}) = JS_{2i}$, $i = 1,2,\ldots,n-1$

$\varphi(x_1') = JS_1$

$\varphi(x_{1+i}') = JS_{1+2i}$, $i = 1,2,\ldots,n-1$

$\varphi(x_{n+1+i}') = JS_{2n+i}$, $i = 0,1,2,\ldots,n-1$

Hence $e_\varphi(0) = 2n-1, e_\varphi(1) = 2n,$

From the above labeling pattern, $|e_\varphi(0) - e_\varphi(1)| \leq 1$.Thus the $P_n \odot C_3$ is Jacobsthal sum cordial labeling, for all $n \geq 2$.

Example 2.8 : Jacobsthal sum cordial of $P_3 \odot C_3$

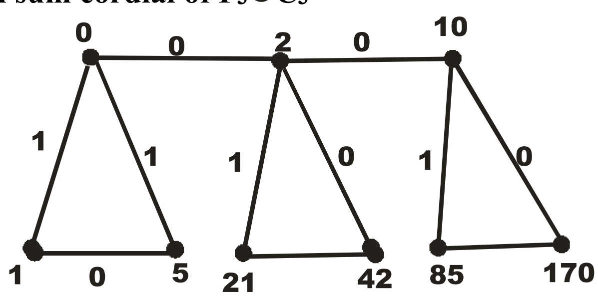

Figure 2.8

Conclusion:

In this paper, we have proved that some of the cycle related graphs, such as Dutch windmill graph D_4^n, crown graph, jewell graph, extended jewell graph, jewell graph without prime edge, globe graph, shadow graph, $P_n \odot C_3$ graphs are jacobsthal sum cordial labeling.

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