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SPILT DUPLEX EQUITABLE DOMINATION NUMBER OF A GRAPH

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Abstract

Let G = (V, E) be a graph. A subset D ⊆ V is said to be a dominating set of G if every vertex in V-D is adjacent to some vertex in D. The cardinality of a minimum dominating set D is called the domination number of and is denoted by γ(G). A duplex equitable dominating set D of a graph G is split duplex equitable dominating set if the induced subgraph $< V - D >$ is disconnected. The minimum cardinality of split duplex equitable dominating set is called split duplex equitable domination number and it is denoted by $\gamma_{sde}(G)$. In this paper, we obtain some results on Split Duplex Equitable domination number of a graph, found relationship between other domination parameters and we found the best possible upper and lower bounds for γ_{sde} then we characterize the graphs satisfies these bounds, obtained γ_{sde} number for some standard graphs.

Key words: equitable, domination number, split domination, duplex

AMS Subject Classification: **05c69**

1.Introduction

The graph considered here are simple, finite, connected, undirected with vertex set V and edge set E. A graph $G = (V, E)$ consists of set of objects $V = \{v_1, v_2, v_3 \ldots\ldots\}$ called vertices (nodes or points) and $E = \{e_1, e_2, e_3 \ldots\ldots.\}$ whose elements are called edges. The vertices are represented as points and each edge as a line segment joining its end vertices. The number of vertices and edges are denoted by m and n respectively. A graph G is called complete graph if every pair of vertices are adjacent in A. The complete graph with n vertices is denoted by K_n. A bipartite graph is a graph whose vertex V (G) can be partitioned two non-empty subset V_1 and V_2 such that every edge of G one end in $V_1$and another in V_2,is called (V_1, V_2) bipartition of G. If further every vertex of V_1 is joined to every vertex of V_2, the G is called complete bipartite graph

10.48047/jocaaa.2024.33.07.94

with partition (V_1, V_2) such that $|V_1|$=s and $|V_2|$= t is denoted by $K_{s,t}$. A degree of a vertex v in a graph G is the number of edges incident with the vertex v and it is denoted by deg (v). The minimum degree and maximum degree of the G denoted by $\delta(G)$ and $\Delta(G)$ respectively. A star, denoted by $K_{1,n-1}$ is a tree with one root vertex and $n-1$ pendant vertex. A wheel is denoted by W_n is a graph with p vertices formed by connecting a single vertex to all vertices of $n-1$ cycles. A graph $G_1$is isomorphic to G_2 if there exists a bisection $\emptyset$ from V (G_1) to V (G_2) such that $uv \in E(G_1)$ if and only if $\emptyset(u)\emptyset(v) \in E(G_2)$. If G_1 is isomorphic to G_2, then we write $G_1 \cong G_2$.For any graph G, the semi total point graph $T_2(G)$ is the graph whose vertex set is the union of vertices and edges in which two vertices are adjacent if and only if they are adjacent vertices of G or one is a vertex and other is an edge of G incident with it. Vertex coloring is an assignment of colors to the vertices of a graph 'G' such that no two adjacent vertices have the same color. The minimum number of colors required for vertex coloring of graph 'G' is called as the chromatic number of G, denoted by χ(G). Let G = (V, E) be a graph. A subset D $\subseteq V$ is said to be a dominating set of G if every vertex in V−D is adjacent to some vertex in D. The minimum cardinality taken for all the dominating set is called the domination number and it is denoted by $\gamma(G)$. A subset D of V is said to be equitable dominating set of G if for every v$\in V-D$ there exists a vertex u $\in D$ such that uv$\in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$. The minimum cardinality such dominating set is called equitable domination number and it is denoted by $\gamma^e(G)$. A dominating set D of a graph G is a non-split dominating set if the induced subgraph <V-D> is connected. The minimum cardinality non-split dominating set is called non-split domination number and it is denoted by $\gamma_{ns}(G)$. In this chapter we introduce the concept of split duplex equitable domination number of a graph. Also split duplex equitable domination number is determined of some general graph and standard graph and its characterization is discussed. Then some upper bounds were obtained.

2.Split duplex equitable domination number

Definition

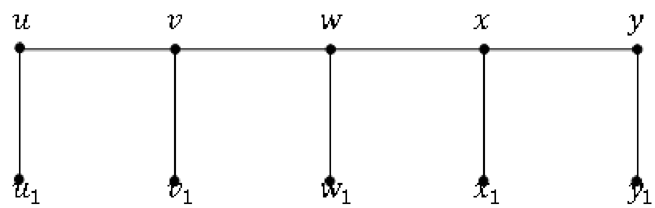
A duplex equitable dominating set D of a graph G is split duplex equitable dominating set if the induced sub graph $< V - D >$ is disconnected. The minimum cardinality of split

10.48047/jocaaa.2024.33.07.94

duplex equitable dominating set is called split duplex equitable domination number and it is denoted by $\gamma_{sde}(G)$.

Example :1

Consider the graph



Here $D = \{u, w, y, u_1, v_1, w_1, x_1, y_1\}$ and $V - D = \{v, x\}$.

Let $v \in V - D$, then $vu, vw \in E(G)$ and $|\deg(v) - \deg(u)| \leq 1$

Let $x \in V - D$, then $xw, xy \in E(G)$ and $|\deg(x) - \deg(w)| \leq 1$ also induced subgraph $< V - D >$ is disconnected. Hence D is split duplex equitable dominating set. Also $\gamma_{sde}(G) = 5$

2 Split Duplex Equitable Domination Number for general and special graphs

Theorem :2.1

For all paths (P_n), $\gamma_{sde}(P_n) = \begin{cases} \frac{n+1}{2} & if\ n\ is\ odd \\ \frac{n}{2} & if\ n\ is\ even \end{cases}$

Proof:

Let $V(P_n) = \{v_1, v_2, \dots \dots, v_n\}$ be the vertices of P_n and $deg(v_1) = deg(v_n) = 1$ and $deg(v_i) = 2, for\ i = 2,3 \dots . n - 1$.

Case(i) If n is odd

10.48047/jocaaa.2024.33.07.94

Take $D = \{v_1, v_3, \dots \dots v_{n-2}, v_n\}$ and $V - D = \{v_2, v_4, \dots \dots v_{n-1}\}$. For any $v_i \in V - D$, v_i is adjacent to v_{i-1} and v_{i+1} also $|deg(v_i) - deg(v_{i+1})| \leq 1$. Clearly $< V - D >$ is disconnected. Hence D is minimum split duplex equitable dominating set. therefore

$\gamma_{sde}(P_n) = \frac{n+1}{2}$

Case(ii) if n is even

The proof similar of case (i)

Theorem :2.2

For all cycle $(C_n), \gamma_{sde}(C_n) = \begin{cases} \frac{n+1}{2} & if\ n\ is\ odd \\ \frac{n}{2} & if\ n\ is\ even \end{cases}$

Proof:

Let V$(C_n) = \{v_1, v_2, \dots \dots, v_n\}$ be the vertices of P_n and $deg(v_i) = 2, for\ i = 1,2,3 \dots . n$.

Case(i) If n is odd

Take $D = \{v_1, v_3, \dots \dots v_{n-2}, v_n\}$ and $V - D = \{v_2, v_4, \dots \dots v_{n-1}\}$. For any $v_i \in V - D$, v_i is adjacent to v_{i-1} and v_{i+1} also $|deg(v_i) - deg(v_{i+1})| \leq 1$. Clearly $< V - D >$ is disconnected. Hence D is minimum split duplex equitable dominating set. Therefore

$\gamma_{sde}(P_n) = \frac{n+1}{2}$

Case(ii) if n is even

The proof similar of case (i)

Theorem :2.3

For complete bipartite graph $(K_{p,q}), \gamma_{sde}\big((K_{p,q}\big) = \min\{p, q\}$ if $|p - q| \leq 1$

Proof:

10.48047/jocaaa.2024.33.07.94

Let V$(K_{p,q}) = \{v_1, v_2, \dots\dots, v_p, u_1, u_2, \dots\dots, u_q\}$ be the vertices of $K_{p,q}$ and $deg(v_i) = q, for\ i = 1,2,3 \dots. p.\ deg(u_i) = p, for\ i = 1,2,3 \dots. q.$

If $p \le q$, Take $D = \{v_1, v_2, \dots\dots, v_p\}$ and $V - D = \{u_1, u_2, \dots\dots, u_q\}$. For every vertices $u_i \in V - D$, u_i is adjacent for any $v_j, v_k \in D$ and $|deg(u_i) - deg(v_j)| = |p - q| \le 1$and Clearly $< V - D >$ is disconnected. Hence D is minimum split duplex equitable dominating set. Therefore $\gamma_{sde}((K_{p,q}) = p$.

If $q \le p$, Take $D = \{u_1, u_2, \dots\dots, u_q\}$ and $V - D = \{v_1, v_2, \dots\dots, v_p\}$. For every vertices $v_i \in V - D$, u_i is adjacent for any $u_j, u_k \in D$ and $|deg(u_i) - deg(v_j)| = |p - q| \le 1$and Clearly $< V - D >$ is disconnected. Hence D is minimum split duplex equitable dominating set. Therefore $\gamma_{sde}((K_{p,q}) = q$.

Theorem : 2.4

For any wheel $(W_{1,n})$, $\gamma_{sde}(W_{1,n}) = \begin{cases} \frac{n+2}{2}\ if\ n\ is\ even \\ \frac{n+3}{2}\ if\ n\ is\ odd \end{cases}$

Proof:

Let V$(W_{1,n}) = \{v, v_1, v_2, \dots\dots, v_n\}$ be the vertices of $W_{1,n}$ also $deg(v) = n$ and $deg(v_i) = 3, for\ i = 1,2,3 \dots. n.$

Case(i) If n is odd

Take $D = \{v, v_2, v_4, \dots\dots v_{n-2}, v_n\}$ and $V - D = \{v_1, v_3, \dots\dots v_{n-1}\}$.

For any $v_i \in V - D$, v_i is adjacent to v_{i-1} and v_{i+1} also $|deg(v_i) - deg(v_{i+1})| \le 1$.

Clearly $< V - D >$ is disconnected. Hence D is minimum split duplex equitable dominating set. Therefore $\gamma_{sde}(W_{1,n}) = \frac{n+2}{2}$

10.48047/jocaaa.2024.33.07.94

Case(ii) if n is even

The proof similar of case (i)

Theorem :2.5

For any Helen graph (H_n), $\gamma_{sde}(H_n) = \begin{cases} \frac{3n+2}{2} \; if\ n\ is\ even \\ \frac{3n+3}{2} \; if\ n\ is\ odd \end{cases}$

Proof:

Let $\mathrm{V}(H_n) = \{v, v_1, v_2, \dots \dots, v_n, v_1', v_2', v_3' \dots., v_n'\}$ be the vertices of H_n also $deg(v) = n$, $deg(v_i) = 4,$ and $deg(v_i') = 1\ for\ i = 1,2,3 \dots. n$

Case(i) If n is odd

Take $D = \{v, v_2, v_4, \dots \dots v_{n-2}, v_n, v_1', v_2', v_3' \dots., v_n'\}$ and $V - D = \{v_1, v_3, \dots \dots v_{n-1}\}$.

For any $v_i \in V - D$, v_i is adjacent to v_i' and v_{i+1} also $|deg(v_i) - deg(v_{i+1})| \leq 1$. Clearly $< V - D >$ is disconnected. Hence D is minimum split duplex equitable dominating set.

Therefore $\gamma_{sde}(H_n) = \frac{3n+2}{2}$

Case(ii) if n is even

The proof similar of case (i)

Theorem :2.6

For any combo graph $(P_n \circ P_2)$, $\gamma_{sde}(P_n \circ P_2) = \begin{cases} \frac{3n}{2} \; if\ n\ is\ even \\ \frac{3n+1}{2} \; if\ n\ is\ odd \end{cases}$

Proof:

Let $\mathrm{V}(P_n \circ P_2) = \{v_1, v_2, \dots \dots, v_n, v_1', v_2', v_3' \dots., v_n'\}$ be the vertices of $P_n \circ P_2$ and $deg(v_1) = deg(v_n) = 2$ and $deg(v_i) = 3, for\ i = 2,3 \dots. n - 1$ and $deg(v_i') = 1\ for\ i = 1,2,3 \dots. n$

10.48047/jocaaa.2024.33.07.94

Case(i) If n is even

Take $D = \{v_1, v_3, \dots \dots v_{n-2}, v_n, v_1', v_2', v_3' \dots ., v_n'\}$ and $V - D = \{v_2, v_4, \dots \dots v_{n-1}\}$. For any $v_i \in V - D$, v_i is adjacent to v_i' and v_{i+1} also $|deg(v_i) - deg(v_{i+1})| \leq 1$. Clearly $< V - D >$ is disconnected. Hence D is minimum split duplex equitable dominating set. therefore $\gamma_{sde}(P_n \circ P_2) = \frac{3n}{2}$

Case(ii) if n is odd

The proof similar of case (i)

Theorem :2.7

For any crown graph $(C_n \circ P_2)$, $\gamma_{sde}(C_n \circ P_2) = \begin{cases} \frac{3n}{2} \; if\ n\ is\ even \\ \frac{3n+1}{2} \; if\ n\ is\ odd \end{cases}$

Proof:

Let V$(C_n \circ P_2) = \{v_1, v_2, \dots \dots, v_n, v_1', v_2', v_3' \dots ., v_n'\}$ be the vertices of $C_n \circ P_2$. Then $deg(v_i) = 3, for\ i = 1,2,3 \dots . n$ and $deg(v_i') = 1\ for\ i = 1,2,3 \dots . n$

Case(i) If n is even

Take $D = \{v_1, v_3, \dots \dots v_{n-2}, v_n, v_1', v_2', v_3' \dots ., v_n'\}$ and $V - D = \{v_2, v_4, \dots \dots v_{n-1}\}$. For any $v_i \in V - D$, v_i is adjacent to v_i' and v_{i+1} also $|deg(v_i) - deg(v_{i+1})| \leq 1$. Clearly $< V - D >$ is disconnected. Hence D is minimum split duplex equitable dominating set. therefore $\gamma_{sde}(C_n \circ P_2) = \frac{3n}{2}$

Case(ii) if n is odd

The proof similar of case (i)

Theorem : 2.8

For any bistar graph $(B_{p,q})$, $\gamma_{sde}(B_{p,q}) = p + q - 1$ if $|p - q| \leq 1$

10.48047/jocaaa.2024.33.07.94

Proof:

Let V$(B_{p,q}) = \{v, v_1, v_2, \dots \dots v_p, u, u_1, u_2, \dots \dots ., u_q\}$ be the vertices of $B_{p,q}$. Then $deg(v) = p, deg(v) = q, deg(v_i) = 1 for\ i = 1,2,3 \dots . p$ and $deg(u_i) = 1\ for\ i = 1,2,3 \dots . q$

Take $D = \{v, v_1, v_2, \dots \dots v_p, u_1, u_2, \dots \dots ., u_q\}$ and $V - D = \{u\}$. For any $u \in V - D$, u is adjacent to u_i and v also $|deg(u) - deg(v)| = |p - q| \leq 1$. Clearly $< V - D >$ is disconnected. Hence D is minimum split duplex equitable dominating set. therefore $\gamma_{sde}(B_{p,q}) = p + q - 1$

Theorem :9

For all book graph $(B_n), \gamma_{sde}(B_n) = n + 1$

Proof:

Let V$((B_n) = \{v, v_1, v_2, \dots \dots v_n, u, u_1, u_2, \dots \dots ., u_n\}$ be the vertices of B_n . Then $deg(v) = n, deg(v) = n, deg(v_i) = 2\ for\ i = 1,2,3 \dots . p$ and $deg(u_i) = 2\ \ for\ i = 1,2,3 \dots . n$.

Take $D = \{v, u_1, u_2, \dots \dots ., u_n\}$ and $V - D = \{u, v_1, v_2, \dots \dots v_n\}$. For any $u \in V - D$, u is adjacent to u_i and v also $|deg(u) - deg(v)| \leq 1$. Clearly $< V - D >$ is disconnected. Hence D is minimum split duplex equitable dominating set. Therefore $\gamma_{sde}(B_n) = n + 1$

Theorem :2.10

For any Fan graph $(F_{1,n})$, $\gamma_{sde}(F_{1,n}) = \begin{cases} \frac{n}{2} & if\ n\ is\ even \\ \frac{n+1}{2} & if\ n\ is\ odd \end{cases}$

Proof:

10.48047/jocaaa.2024.33.07.94

Let $V(F_{1,n}) = \{v, v_1, v_2, \dots v_n,\}$ be the vertices of $F_{1,n}$. Then $deg(v) = n, deg(v_1) = 2$, $deg(v_n) = 2$,and $deg(v_i) = 3 \ \ for\ i = 2,3 \dots . n - 1$

Case(i) If n is even

Take $D = \{v, v_1, v_3, \dots v_{n-1}\}$ and $V - D = \{v_2, v_4, \dots \dots v_n\}$. For any $v_i \in V - D$, u is adjacent to v_{i+1} and v also $|deg(v_i) - deg(v_{i+1})| \leq 1$. Clearly $< V - D >$ is disconnected. Hence D is minimum split duplex equitable dominating set. Therefore

$\gamma_{sde}(F_{1,n}) = \frac{n}{2}$

Case(ii) if n is odd

The proof similar of case (i)

Theorem :2.11

For any Fan graph $(F_{1,n}{}^\circ P_2)$, $\gamma_{sde}\left(F_{1,n}{}^\circ P_2\right) = \begin{cases} \frac{3n}{2}\ if\ n\ is\ even \\ \frac{3n+1}{2}\ if\ n\ is\ odd \end{cases}$

Proof:

By theorem: 10 we know that $\gamma_{sde}\left(F_{1,n}\right) = \begin{cases} \frac{n}{2}\ \ if\ n\ is\ even \\ \frac{n+1}{2}\ \ \ if\ n\ is\ odd \end{cases}$

The graph $F_{1,n}{}^\circ P_2$ obtained by adding a P_2 to each vertex of $F_{1,n}$. That means add a pendent vertex to each vertex of $F_{1,n}$. Every duplex equitable dominating set contains the pendent vertices. So add all pendent vertices of minimum split duplex equitable dominating set of $F_{1,n}$.

If n is even then $\gamma_{sde}\left(F_{1,n}{}^\circ P_2\right) = \frac{n}{2} + n = \frac{3n}{2}$

If n is odd then $\gamma_{sde}\left(F_{1,n}{}^\circ P_2\right) = \frac{n+1}{2} + n = \frac{3n+1}{2}$

10.48047/jocaaa.2024.33.07.94

Theorem :2.12

For any friendship graph (C_3^t) , $\gamma_{sde}(C_3^t) = t + 1$

Proof:

Let $V(C_3^t) = \{v, a_1, b_1, a_2, b_2, \dots \dots, a_t, b_t\}$ be the vertices of C_3^t . Then $deg(v) = t, deg(a_i) = 2 \; and \; deg(b_i) = 2 \;\; for \; i = 1,2,3 \dots . t$

Take $D = \{v, a_1, a_2, \dots \dots, a_t\}$ and $V - D = \{b_1, b_2, \dots \dots, b_t\}$. For any $b_i \in V - D$, b_i is adjacent to v and a_i also $|deg(b_i) - deg(a_i)| \leq 1$. Clearly $< V - D >$ is disconnected. Hence D is minimum split duplex equitable dominating set. therefore $\gamma_{sde}(C_3^t) = t + 1$

3. Bounds of split duplex equitable dominating set

Observation 3.1

Let G be a non-complete graph then there exists two vertices u and which is not adjacent. If u and v are equitable vertices then $|\deg(u) - \deg(x)| \leq 1$ for all $x \in N(u)$ and $|\deg(v) - \deg(y)| \leq 1$ for all $y \in N(v)$. Then $G - \{u, v\}$ cannot be complete then there exists a vertex x is adjacent to u with $deg\,(x) \geq n - 2$. Hence degree $= n - 1$ this implies that u adjacent v which is contradiction. Therefore there exists $x, y \in G - \{u, v\}$ which is not adjacent.

Remark:3.1

The split duplex equitable dominating set exists if $\delta(G) \geq 2$

Assume that there are vertices which is not equitable and sub graph induced by set of all not equitable vertices is not complete or contains at least two non-trivial components.

10.48047/jocaaa.2024.33.07.94

Remark:3.2

Suppose G is disconnected graph contains a minimum split duplex equitable dominating set. Then any minimum split duplex equitable dominating sets is a minimum split duplex equitable dominating set.

Observation :3.2

k(G) ≤ $\gamma_{sde}(G)$, where k(G) is the connectivity of G

Theorem :3.1

For any graph G, $\gamma(G) \leq \gamma_{de}(G) \leq \gamma_{sde}(G)$

Proof:

Every split duplex equitable dominating set is duplex equitable dominating set also every duplex equitable dominating set is a dominating set. Hence $\gamma(G) \leq \gamma_{de}(G) \leq \gamma_{sde}(G)$.

Theorem :3.2

For any graph, $\gamma_{de}(G) \leq \min\{\gamma_{nsde}(G), \gamma_{sde}(G)\}$

Proof:

Since every split duplex equitable dominating set and non split duplex equitable dominating set of G are the duplex equitable dominating set of G we have $\gamma_{de}(G) \leq \gamma_{sde}(G)$ and $\gamma_{de}(G) \leq \gamma_{nsde}(G)$. Hence $\gamma_{de}(G) \leq \min\{\gamma_{nsde}(G), \gamma_{sde}(G)\}$

Observation :3.3

For any connected graph, $2 \leq \gamma_{sde}(G) \leq n-2$

Observation :3.4

If H is a connected spanning sub graph of G then $\gamma_{sde}(H) \leq \gamma_{sde}(G)$

10.48047/jocaaa.2024.33.07.94

Theorem :3.3

A duplex equitable dominating set D of G is split duplex equitable dominating set if and only if there exists two vertices v_1 and v_2 in $V-D$ such that v_1-v_2 path contains a vertex of D.

Proof:

Suppose D is duplex equitable dominating set and if there exists v_1 and v_2 in $V-D$ such that v_1-v_2 path contains a vertex of D. then $< V-D >$ is not connected. Then D is split duplex equitable dominating set.

Conversely

Suppose $< V-D >$ is disconnected. So take v_1 and v_2 in different components of $V-D$. Then any v_1-v_2 path contains a vertex of D.

Theorem :3.4

A split duplex equitable dominating set is minimal if and only if for each vertex $v \in D$ one of the following holds

(i) There exists a vertex $u \in V-D$such that $N(u) \cap D = \{v\}$ or $N^e(u) \cap D = \{v\}$

(ii) v is equitable strong in $< D >$

(iii) $\big((V-D) \cup \{v\}\big)$ is connected

Proof :

Suppose D is minimal split duplex equitable dominating set such that v does not stratify any of the above conditions. Suppose (i) and (ii) not satisfy then D is not duplex equitable dominating set hence $D-\{v\}$ duplex equitable dominating set. Suppose (iii) is not satisfy $\big((V-D) \cup \{v\}\big)$ is disconnected. Therefore $D-\{v\}$ split duplex equitable

10.48047/jocaaa.2024.33.07.94

dominating set which is a contradiction to D is minimal split duplex equitable dominating set . Hence v is satisfy one of the above conditions.

Converse is obious

Remark:3.3

Any super set D of split duplex equitable dominating set is a split duplex equitable dominating set if $|D| \leq n-2$.

Theorem :3.5

$$\gamma_{sde}(G) \leq \frac{n\Delta(G)}{\Delta(G)+1}$$

Proof:

Let D is minimal split duplex equitable dominating set. Since D is minimal for any vertex $v \in D$ $\left((V-D) \cup \{v\}\right)$ is connected. Therefore there exists a vertex $u \in V-D$ such that u is adjacent to v. so $V-D$ is a dominating set. Since $|V-D| \geq 2$, $V-D$ duplex equitable dominating set

Therefore $\gamma_{de}(G) \leq |V-D| \leq n-\gamma_{sde}(G)$

But $\frac{n}{\Delta(G)+1} \leq \gamma(G) \leq \gamma_{de}(G) \leq \gamma_{sde}(G)$

Hence $\gamma_{sde}(G) \leq n-\gamma_{de}(G)$

$$\leq n-\frac{n}{\Delta(G)+1}$$

$$=\frac{n\Delta(G)}{\Delta(G)+1}$$

So $\gamma_{sde}(G) \leq \frac{n\Delta(G)}{\Delta(G)+1}$

Theorem :3.6

10.48047/jocaaa.2024.33.07.94

For any graph G, $n-\frac{m}{2}+\frac{m_0}{2} \leq \gamma_{sde}(G)$ where G is (n,m) graph and $m_0 = \min q(< D >)$ where D is minimal split duplex equitable dominating set.

Proof:

Let D be minimal split duplex equitable dominating set of G. Then for every $v \in V - D$,there exists atleast two edges from u in D and there exists at least $2|V-D|$ edges from $V-D$ to D. Then number of edges of G, $m > 2|V-D| + m_0$

$$m > 2|V| - 2|D| + m_0$$

$$2|D| > 2|V| - m + m_0$$

$$2\gamma_{sde}(G) > 2n - m + m_0$$

$$\gamma_{sde}(G) > n - \frac{m}{2} + \frac{m_0}{2}$$

Hence $n-\frac{m}{2}+\frac{m_0}{2} \leq \gamma_{sde}(G)$

Theorem :3.7

For any graph G, $\gamma_{sde}(G) \geq \left\lceil\frac{2n}{\Delta+2}\right\rceil$

Proof:

Every vertex in $V-D$ contribute two to the degree of sum of vertices of D. since $2|V-D| \leq \sum \deg(u)$ for u$\in D$ where D is split duplex equitable dominating set of G. therefore $2|V-D| \leq \sum \deg(u) \leq \gamma_{sde}(G)\Delta$ which implies $2|D| - 2|V| \leq \gamma_{sde}(G)\Delta$ which means $2n - 2\gamma_{sde}(G) \leq \gamma_{sde}(G)\Delta$ and $\gamma_{sde}(G)(\Delta+2)$ ≥2n

Hence $\gamma_{sde}(G) \geq \left\lceil\frac{2n}{\Delta+2}\right\rceil$.

Theorem : 3.8

10.48047/jocaaa.2024.33.07.94

For any connected graph G, $\gamma_{sde}(G) + k(G) \leq n + \Delta(G) - 2$. The bound is sharp if $G \cong C_4$

Proof:

For any connected graph G, $\gamma_{sde}(G) \leq n - 2$ and $k(G) \leq \Delta(G)$ so that $\gamma_{sde}(G) + k(G) \leq n + \Delta(G) - 2$. The equality hold if $\gamma_{sde}(G) = n - 2$ and $k(G) = \Delta(G)$. If $G \cong C_4$ then $\gamma_{sde}(G) = 2$ and $k(G) = 2$. Hence the bound is sharp.

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