10.48047/jocaaa.2023.31.01.43

Dom-Chromatic Number of Grid Graphs

Dr. S. Anusha

Assistant Professor, Department of Mathematics, S.T.Hindu college, Nagercoil-629002, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India.
E-mail: anusha.s.thaya@gmail.com

Received: 10.11.2022 Revised: 20.12.2022 Accepted: 10.1.2023

Abstract

Let G be a graph. For a given γ-coloring of a graph G, a dominating set $S \subseteq V(G)$ is said to be a dom-coloring set if it contains atleast one vertex of each color class G. The dom-chromatic number of graph G is the minimal cardinality taken over all its dom-coloring sets and is denoted by $\gamma_{d-c}(G)$[2]. In this paper, we obtain dominator chromatic number of grid graphs.

Key words: *Dominator chromatic number, grid graph.*

Introduction

All graphs considered in this paper are finite, undirected graphs and we follow standard definition of graph theory as found in [1]. Let G = (V, E) be a graph of order n. The open neighborhood N(v) of a vertex v ∈ V(G) consists of the set of all vertices adjacent to v. The closed neighborhood of v is N[v] =N (v) ∪ {v}. The path and cycle of order n are denoted by P_n and C_n respectively. For any two graphs G and H, we define the cartesian product, denoted by G×H, to be the graph with vertex set V(G)×V(H) and edges between two vertices (u_1, v_1) and (u_2, v_2) if either $u_1=u_2$ and $v_1v_2 \in E(H)$ or $u_1u_2 \in E(G)$ and $v_1=v_2$. A grid graphs can be defined as $P_m \times P_n$ were, m, n ≥2 and denoted by $P_{m\times n}$.

A subset S of V is called a dominating set if every vertex in V-S is adjacent to at least one vertex in S. The dominating set is minimal dominating set if no proper subset of S is a dominating set of G. The domination number γ is the minimum cardinality taken over all minimal dominating set of G. A γ-set is any minimal dominating set with cardinality γ.

A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The minimum number of colors for which there exists a proper

10.48047/jocaaa.2023.31.01.43

coloring of G is called chromatic number of G and is denoted by γ(G).For a given γ-coloring of a graph G ,a dominating set $S \subseteq V(G)$ is said to be a dom-coloring set if it contains atleast one vertex of each color class of G.The dom-chromatic number of graph G is the minimal cardinality taken over all its dom-coloring sets and is denoted by $\gamma_{d-c}(G)[2]$.

In a proper coloring C of G, a color class of C is a set consisting of all those vertices assigned the same color. Let C ^{I} be a minimal dominator coloring of G. We say that a color class c_i ∈ C ^{I} is called a non-dominated color class (n-d color class) if it is not dominated by any vertex of G. These color classes are also called repeated color classes.

The dominator chromatic number of paths found in [2].

We have the following observation from [2].

Theorem A [2] The path P_n of order $n \geq 2$ has

$$\gamma_{d-c}(p_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil + 1 & if\ n = 2,3,4,5,7 \\ \left\lceil \frac{n}{3} \right\rceil + 2 & otherwise \end{cases}$$

In this paper, we obtain the least value for dominator chromatic number for grid graphs.

Main Results

Theorem 1: If m and n both even then $\gamma_{d-c}(P_{m \times n})$

$$= \begin{cases} \frac{mn}{4} & if\ m,n \equiv 0 (mod 4) \\ \frac{mn}{4} + 1 & otherwise \end{cases}$$

Proof: Let the vertex set of $P_{m \times n} = V(P_{m \times n}) = \left\{ u_{ij} / \begin{smallmatrix} 1 \le i \le m \\ 1 \le j \le n \end{smallmatrix} \right\}$.We consider two cases.

Case (1): Let $m, n \equiv 0 (mod\ 4)$.

Let $D^{(1)} = \left\{ \left\{ u_{ij} / \begin{smallmatrix} i=1,5,9,---------,(m-3) \\ j=2,6,10,---------,(n-2) \end{smallmatrix} \right\} \cup \left\{ u_{ij} / \begin{smallmatrix} i=2,6,10-----,(m-2) \\ j=4,8,12,---------,n \end{smallmatrix} \right\} \cup \left\{ u_{ij} / \begin{smallmatrix} i=3,7,11,--------,(m-1) \\ j=1,5,9,---------,(n-3) \end{smallmatrix} \right\} \cup \left\{ u_{ij} / \begin{smallmatrix} i=4,8,12,---------,m \\ j=3,7,11,--------,(n-1) \end{smallmatrix} \right\} \right\}$ be any arbitrary γ -set of $P_{m \times n}$ .We assign two color 1,2 respectively. So $\gamma_{d-c}(P_{m \times n}) = \frac{mn}{4} + 2$.

10.48047/jocaaa.2023.31.01.43

Case (2): Let $except\ m, n \equiv 0(mod\ 4)$. we have three sub cases.

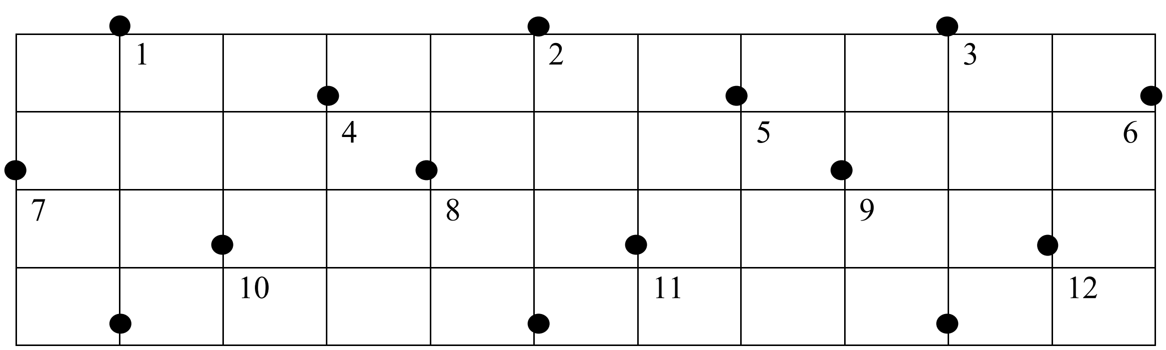
Subcase(2.1): Let $m \equiv 0\ (mod\ 4)$ & $n \equiv 2\ (mod\ 4)$.Let $S^{(1)} = \left\{ \left\{ u_{ij} / {}^{i=1,5,9,--------,(m-3)}_{j=2,6,10,----------,n} \right\} \cup \left\{ u_{ij} / {}^{i=2,6,10--------,(m-2)}_{j=4,8,12,--------,(n-2)} \right\} \cup \left\{ u_{ij} / {}^{i=3,7,11,-------,(m-1)}_{j=1,5,9,--------,(n-1)} \right\} \cup \left\{ u_{ij} / {}^{i=4,8,12,--------,m}_{j=3,7,11,-------,(n-3)} \right\} \cup \{u_{mn}\} \right\}$ be any arbitrary γ -set of $P_{m\times n}$ .We assign two color 1,2 respectively. So $\gamma_{d-c}(P_{m\times n}) = \frac{mn}{4} + 3$.

Subcase(2.2): Let $m \equiv 2\ (mod\ 4)$ & $n \equiv 0\ (mod\ 4)$.Let $S^{(2)} = \left\{ \left\{ u_{ij} / {}^{i=1,5,9,--------,(m-1)}_{j=2,6,10,-------,(n-2)} \right\} \cup \left\{ u_{ij} / {}^{i=2,6,10--------,m}_{j=4,8,12,--------,n} \right\} \cup \left\{ u_{ij} / {}^{i=3,7,11,-------,(m-3)}_{j=1,5,9,--------,(n-3)} \right\} \cup \left\{ u_{ij} / {}^{i=4,8,12,--------,(m-2)}_{j=3,7,11,---------,(n-1)} \right\} \cup \{u_{m1}\} \right\}$ be any arbitrary γ -set of $P_{m\times n}$ .We assign two color 1,2 respectively. So $\gamma_{d-c}(P_{m\times n}) = \frac{mn}{4} + 3$.

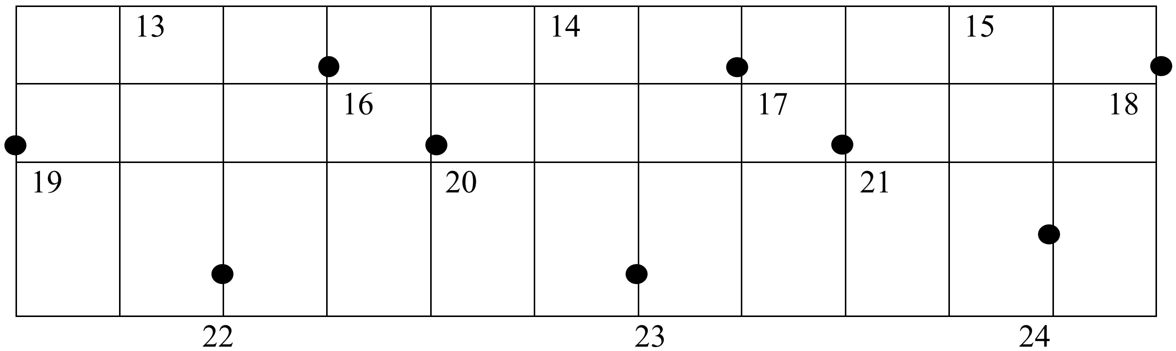
Subcase(2.3): Let $m \equiv 2\ (mod\ 4)$ & $n \equiv 2(mod\ 4)$.Let $S^{(3)} = \left\{ \left\{ u_{ij} / {}^{i=1,5,9,--------,(m-1)}_{j=2,6,10,----------,n} \right\} \cup \left\{ u_{ij} / {}^{i=2,6,10----------,m}_{j=4,8,12,--------,(n-2)} \right\} \cup \left\{ u_{ij} / {}^{i=3,7,11,-------,(m-3)}_{j=1,5,9,--------,(n-1)} \right\} \cup \left\{ u_{ij} / {}^{i=4,8,12,--------,(m-2)}_{j=3,7,11,-------,(n-3)} \right\} \cup \{u_{m1}\} \right\}$ be any arbitrary γ -set of $P_{m\times n}$ .We assign two color 1,2 respectively. So $\gamma_{d-c}(P_{m\times n}) = \frac{mn}{4} + 3$.

Thus $\gamma_{d-c}(P_{m\times n}) = \begin{cases} \frac{mn}{4} & if\ m, n \equiv 0(mod 4) \\ \frac{mn}{4} + 1 & otherwise \end{cases}$ □

Illustration: Consider $\mathbf{P}_{8\times 12}$

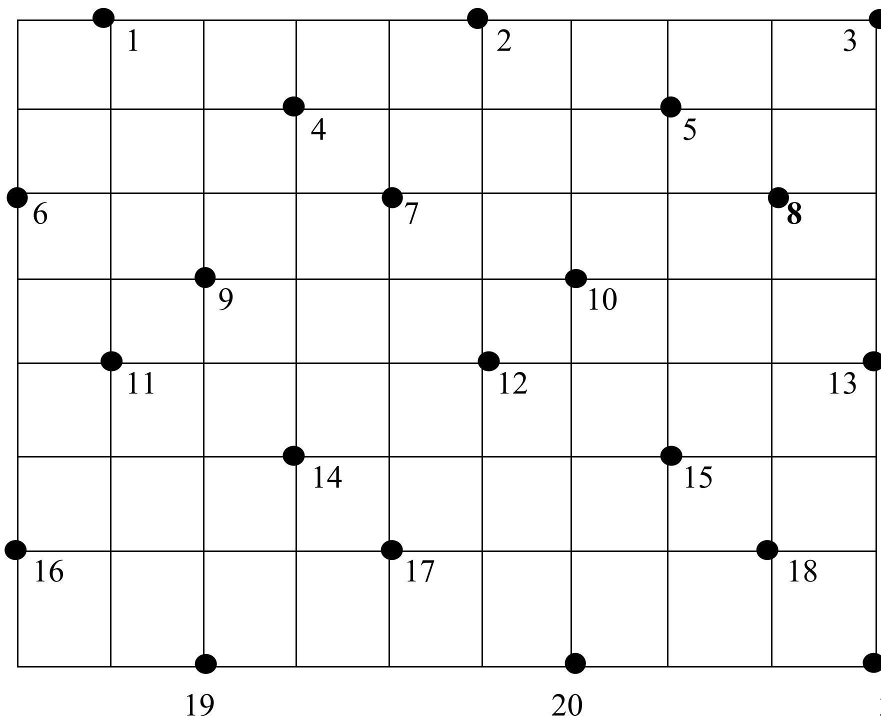


10.48047/jocaaa.2023.31.01.43



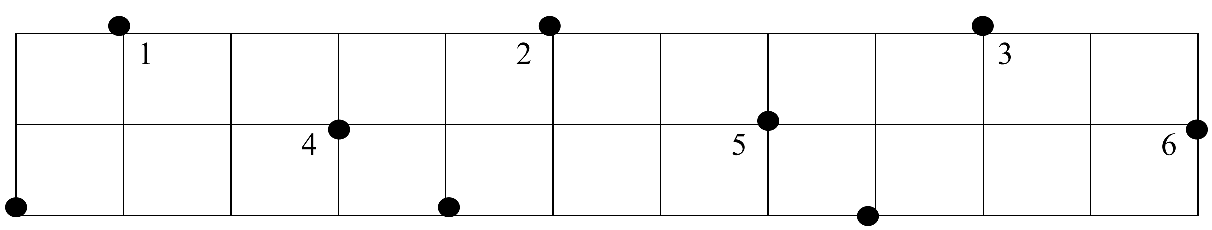
γ_{d-c} ($P_{8\times 12}$) = 24. Fig. 1

Consider $\mathbf{P}_{8\times 10}$

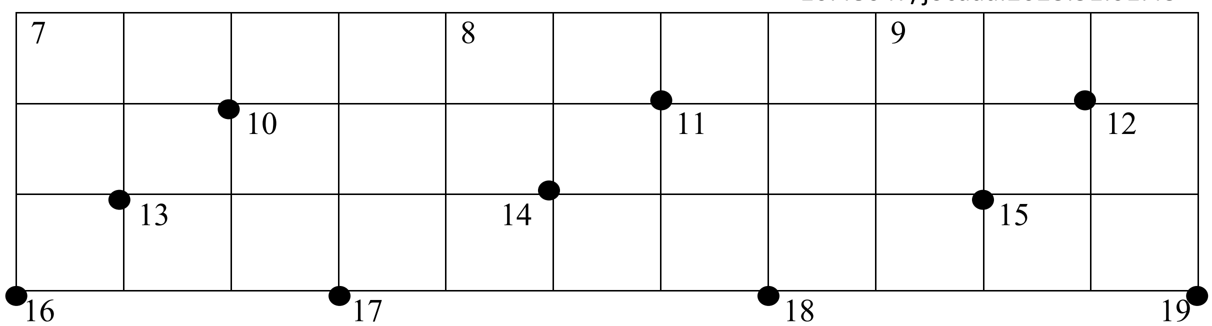


γ_{d-c} ($P_{8\times 10}$) = 21. Fig.2

Consider $\mathbf{P}_{6\times 12}$

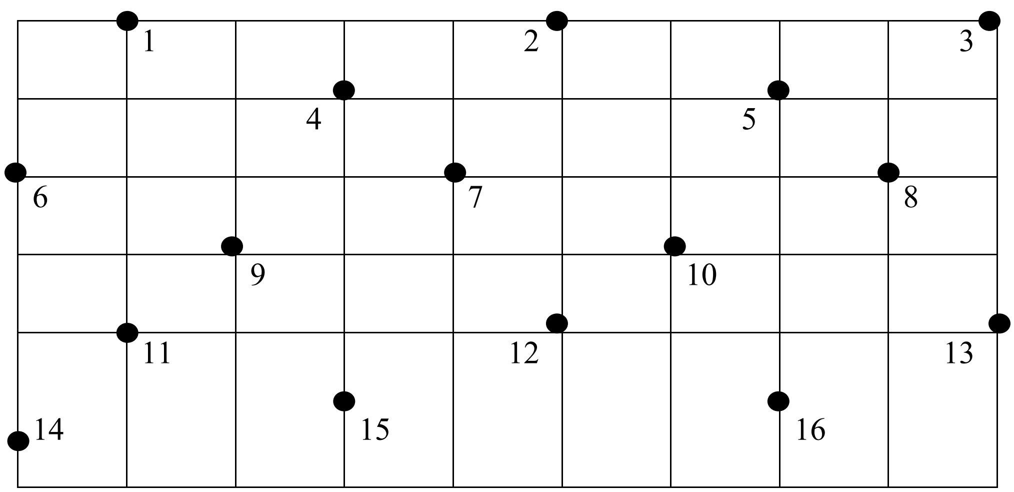


10.48047/jocaaa.2023.31.01.43



γ_{d-c} ($P_{6\times12}$) = 19 Fig.3

Consider $\mathbf{P}_{6\times 10}$



γ_{d-c}

($P_{6\times10}$) = 16 Fig.4

Theorem 2: If m is odd and n is even then

$$\gamma_{d-c}(P_{m\times n}) = \begin{cases} \frac{(m-1)n}{4} + \left\lceil \frac{n}{3} \right\rceil & if\ m-1, n \equiv 0 (mod\ 4) \\ \frac{(m-1)n}{4} + \left\lceil \frac{n}{3} \right\rceil + 1 & otherwise \end{cases}$$

Proof: We have $P_{m\times n}$ is obtained by $P_{(m-1)\times n}$ followed by P_n. Since in a dominator coloring of $P_{m\times n}$ we cannot use the non-repeated colors of vertices in P_n and we can use the same repeated colors of vertices in the graphs $P_{(m-1)\times n}$ and P_n. Since $m-1$ is even, we get by theorem 1,

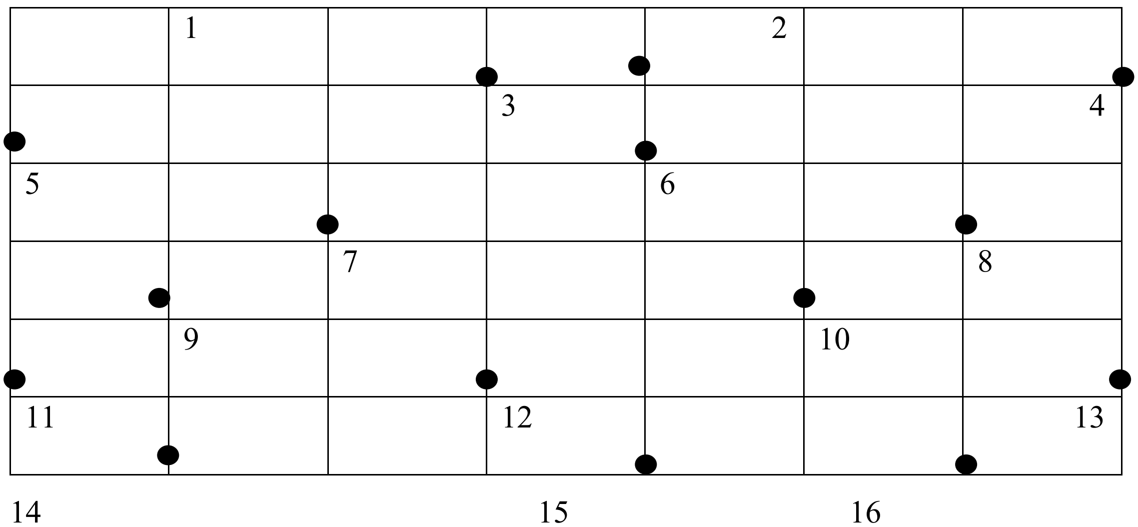
$$\gamma_{d-c}(P_{(m-1)\times n}) = \begin{cases} \frac{(m-1)n}{4} & if\ m-1, n \equiv 0 (mod\ 4) \\ \frac{(m-1)n}{4} + 1 & otherwise \end{cases}$$

10.48047/jocaaa.2023.31.01.43

So $\gamma_{d-c}(P_{m\times n}) = \begin{cases} \frac{(m-1)n}{4} + \left\lceil \frac{n}{3} \right\rceil & if\ m-1, n \equiv 0 (mod\ 4) \\ \frac{(m-1)n}{4} + \left\lceil \frac{n}{3} \right\rceil + 1 & otherwise \end{cases}$ □

Illustration:

Consider $P_{7\times 8}$



$\gamma_{d-c}(P_{7\times 8}) = 16.$ Fig.5

Theorem 3: If m is even and n is odd then

$\gamma_{d-c}(P_{m\times n}) = \begin{cases} \frac{m(n-1)}{4} + \left\lceil \frac{m}{3} \right\rceil & if\ m, n-1 \equiv 0 (mod\ 4) \\ \frac{m(n-1)}{4} + \left\lceil \frac{m}{3} \right\rceil + 1 & otherwise \end{cases}$

Proof: Since m and $n-1 \equiv 0(mod\ 4)$ and $P_{m\times n}$ is obtained by $P_{m\times(n-1)}$ followed by P_m.

By theorem 2, $\gamma_{d-c}(P_{m\times n}) = \gamma_{d-c}(P_{m\times(n-1)}) + \gamma_{d-c}(P_m) - 2$.

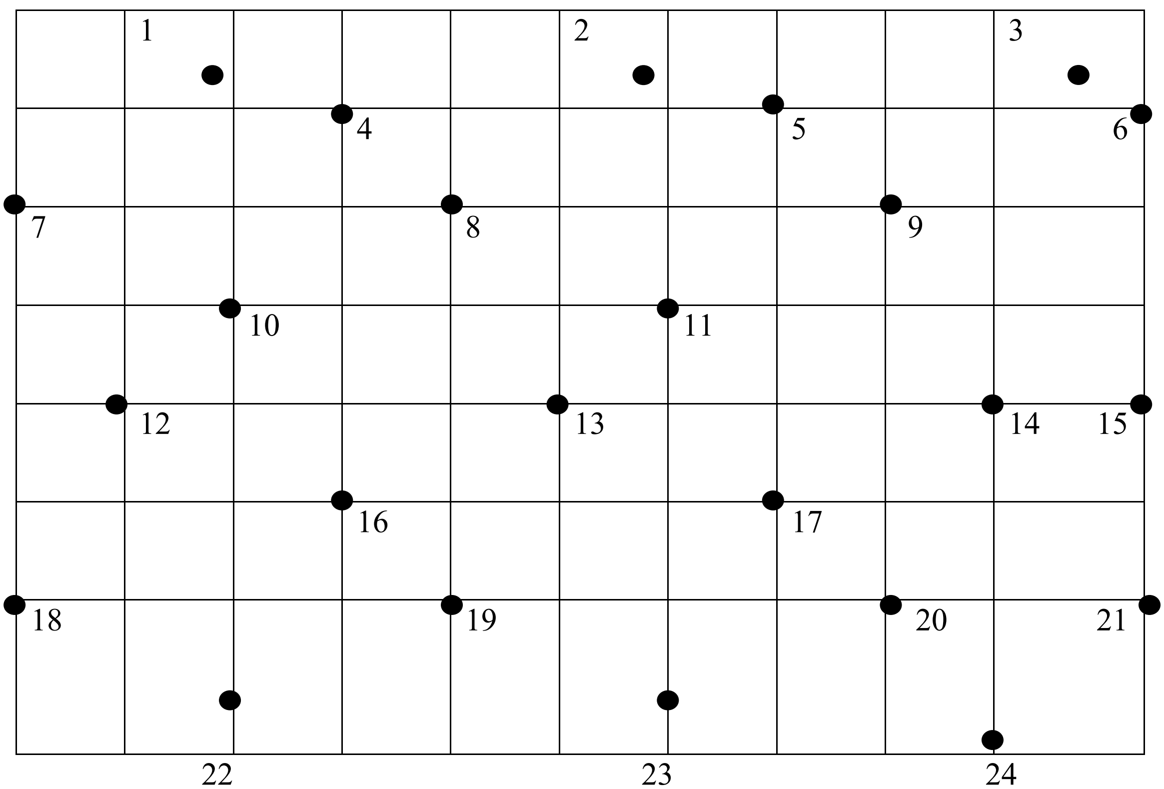
By theorem 1, $\gamma_{d-c}(P_{m\times(n-1)}) = \begin{cases} \frac{m(n-1)}{4} & if\ m, n-1 \equiv 0 (mod\ 4) \\ \frac{m(n-1)}{4} + 1 & otherwise \end{cases}$

So $\gamma_{d-c}(P_{m\times n}) = \begin{cases} \frac{m(n-1)}{4} + \left\lceil \frac{m}{3} \right\rceil & if\ m, n-1 \equiv 0 (mod\ 4) \\ \frac{m(n-1)}{4} + \left\lceil \frac{m}{3} \right\rceil + 1 & otherwise \end{cases}$ □

10.48047/jocaaa.2023.31.01.43

Illustration:

Consider $P_{8\times 11}$



$\gamma_{d-c}(P_{8\times 11}) = 24.$

Fig.5

Theorem 4: If m is odd and n is odd then

$$\gamma_{d-c}(P_{m\times n}) = \begin{cases} \frac{(m-1)(n-1)}{4} + \left\lceil \frac{m+n-1}{3} \right\rceil & if\ m-1, n-1 \equiv 0(mod\ 4) \\ \frac{(m-1)(n-1)}{4} + \left\lceil \frac{m+n-1}{3} \right\rceil + 1 & otherwise \end{cases}$$

Proof: Since (m-1) and (n-1)$\equiv 0(mod\ 4)$ and $P_{m\times n}$ is obtained by $P_{(m-1)\times(n-1)}$ followed by P_{m+n-1}.

By theorem 2, $\gamma_{d-c}(P_{m\times n}) = \gamma_{d-c}(P_{(m-1)\times(n-1)}) + \gamma_{d-c}(P_{m+n-1})$ - 2

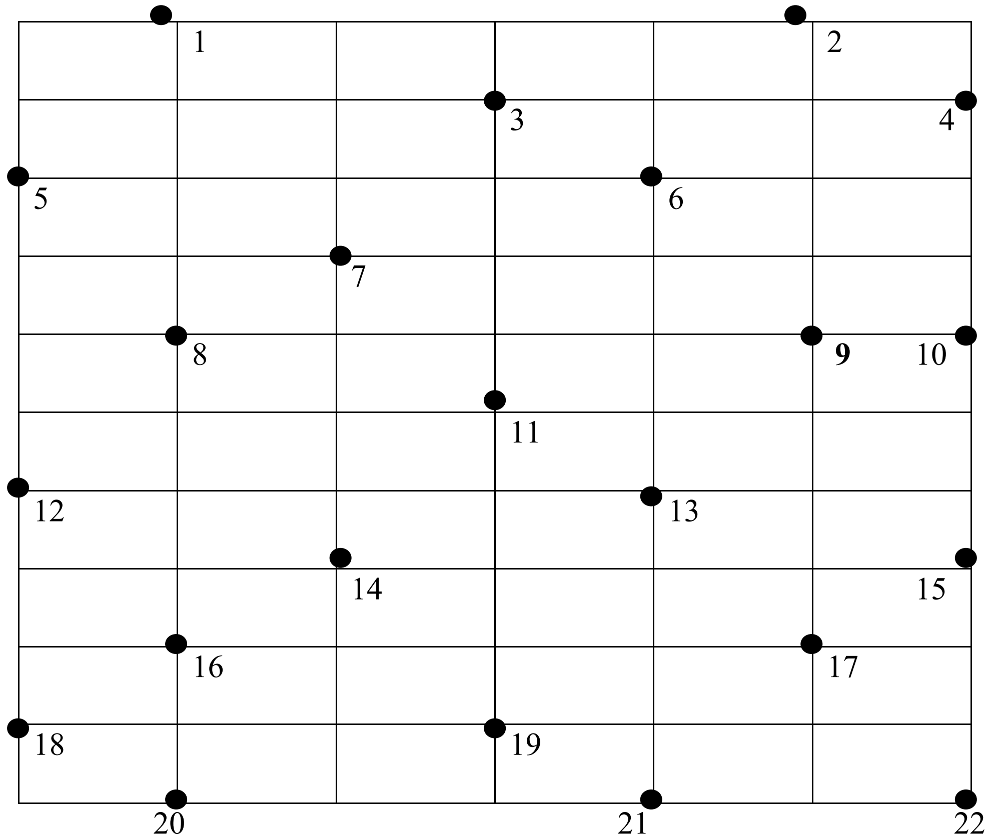
By theorem 1, $\gamma_{d-c}(P_{(m-1)\times(n-1)}) = \begin{cases} \frac{(m-1)(n-1)}{4} & if\ m-1, n-1 \equiv 0(mod\ 4) \\ \frac{(m-1)(n-1)}{4} + 1 & otherwise \end{cases}$

So $\gamma_{d-c}(P_{m\times n}) = \begin{cases} \frac{(m-1)(n-1)}{4} + \left\lceil \frac{m+n-1}{3} \right\rceil & if\ m-1, n-1 \equiv 0(mod\ 4) \\ \frac{(m-1)(n-1)}{4} + \left\lceil \frac{m+n-1}{3} \right\rceil + 1 & otherwise \end{cases}$ □

10.48047/jocaaa.2023.31.01.43

Illustration:

Consider $P_{11\times7}$



γ_{d-c} ($P_{11\times7}$) = 22. Fig.7

References:

[1] F. Harrary , Graph Theory , Addition- wesley Reading, Mass, 1969.

[2] Gera-R, Rasmussen-c and Horton-S, Dominator coloring and safe clique parti tions, congr. Numer 181(2006) 19-32.

[3] S.M.Dedetniemi, S.T.Hedetniemi , A.A.Mcrae ,J.R.S Blair,Dominator coloring of graphs,2006 (pre print).

[4] Terasa W.Haynes,Stephen T.Hedetniemi ,Peter J.Slater, Domination in Graphs, Marcel Dekker,NewYork,1998.

[5] Terasa W.Haynes,Stephen T.Hedetniemi ,Peter J.Slater, Domination in Graphs – Advanced Topics, Marcel Dekker,NewYork,1998

10.48047/jocaaa.2023.31.01.43

[6] S.Gravier, M. Mollard, on domination numbers of Cartesian product of paths, Discrete Appl. Math. 80 (1997) 247-250.

[7] T.Y.Chang , W.E.Clark ,The domination number of the 5×n and 6×n grid graphs, J. Graph Theory 17(1) (1993) 81-107.

[8] M.S.Jacobson, L.F.Kinch, on the domination number of products of a graph I, Ars combin. 10 (1983) 33-44.

[9] B. Chaluvaraju and C. AppajiGowda, Dom-chromatic number of a graph, Malaya Journal of Mathematik, 4(1)(2016), 1–7.

[10] T.N Janakiraman and M. Poobalaranjani, Dom-chromatic sets in Bi-partite graphs, International Journal of Engineering Science, Advanced Computing and Bio-Technology, 1(2)(2010), 80–95.