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THE BRANCHED-PRISM GRAPH $(C_m \times P_3)\odot \bar{K}_n$ AND ITS SUPER (a,d)-EDGE ANTIMAGIC LABELING

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ABTRACT

$Azizu[1]\, et\ al\ defined$ the branched prism graph, denoted by $H_2 = (C_m \times P_2)\odot \bar{K}_n$ for odd $m \geq 3, r \geq 2$, where $\bar{K}_n$ $denotes\ the\ complement\ of\ a\ complet\ graph\ K_n\ on\ n\ isolated$ vertices and they also determined the existence of a super edge magic labeling of the branched –prism graph H_2 . Further $Azizu\ et\ al$ [2] $determined\ that$ H_2 admits a super (a ,d) edge antimagic total lebeling of $H_2 = (C_m \times P_2)\odot \bar{K}_n$

$m \geq 3, r \geq 2. In\ this\ paper\ it\ will\ be\ shown\ that$ $H_3 = (C_m \times P_r)\odot \bar{K}_n$ $m \geq 3, r \geq 3$,admits a super (a, d) edge antimagic total lebeling for odd $m \geq 3, r \geq 3$.

1.INTRODUCTION

All graphs considered in this paper are simple, finite, connected and undirected. We follow the basic notations and terminologies of graph theory as in [7, 14]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Several types of labeling are of interest. A detailed survey of graph labeling can be found in [5]

A Labeling of a graph G is any mapping that sends a certain set of graph elements to a certain set of positive integers. If the domain is the vertex set or the edge set respectively, the labeling is called a vertex labeling or edge labeling respectively. If the domain is the vertex set and the edge set then the labeling is called a total labeling of G. In particular a graph G has vertex set V(G) and edge set E(G) and we let | V(G)|=p and |E(G)|=q. Given a graph G, let V= V(G), E= E(G). A one-one correspondence f from VUE onto integers {1,2,…, p+q} is an edge magic labeling if there is a constant k so that for every edge

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xy, f(x)+ f(xy)+ f(y)=k, the constant k is called the edge magic number. Moreover f is called a super edge magic labeling if V(G)= {1,2,…,p}. Thus, a super edge magic graph is a graph that admits a super edge magic labeling. The above first two definitions were introduced by Kotzig and Rosa[8,9] in 1970 and Enomoto, Llado, Nakamigawa and Ringel [4] in 1998 respectively. It is worth while to mention that Kotzig and Rose called magic valuations, the current term is due to Ringel[10]. Moreover, Simanjuntak et al [11] defined a super (a,d)edge antimagic total labeling of G as a mapping f:VUE→{1,2,…….,p+q} such that the set of weights f(u)+ f(uv)+ f(v) for every edge uv form an arithmetic progression {a, a+d, ……,a+(q1)d}, for integers a>0,d≥ 0. If d=0 then the super (a,o)- antimagic labeling becomes the super magic labeling with magic constant (=k).Some previous results on magic and antimagic labeling are listed in a book by Baca and Miller[3].

It was shown in [13] that the Cartesian prodect H_3=C_nxP_r admits an edge magic total labeling for odd n. Sugeng et al [12] Established the super (a,d) edge antimagic total labeling of a generalized prism graph C_mx P_r ,$m \geq 3, r \geq 2$.$Azizu[1]$ $et\ al\ defined$ the branched prism graph, denoted by $H_2 = (C_m \text{x} P_2) \odot \bar{K}_n$ $m \geq 3, r \geq 2$,where $\bar{K}_n$ $denotes\ the\ complement\ of\ a\ complete\ graph\ on\ n\ isolated$ vertices and they also determined the existence of a super edge magic labeling of the branched –prism graph H_2 . Further $Azizu\ et\ al\ [2]\ determined\ that$ H_2 admits a super (a,d) edge antimagic total lebeling of odd $m \geq 3, r \geq 2$. $In\ this\ paper\ it\ will\ be\ show\ that$ $H_3 = (C_m \text{x} P_3) \odot \bar{K}_n$ admits a super (a, d) edge antimagic total lebeling for odd $m \geq 3, r \geq 3$.

2. The Branched-Prism Graph (C_m xP_3)$\odot \bar{K}_n$ and Its Super (a, d)-Edge Antimagic Labeling In this section we gave the definition of branched-prism graph as follows.

The graph is constructed from the corona operation between prism graph C_m xP_3 and the complement of a complete graph $\bar{K}_n$ on n vertices, denoted by H_3= (C_m×P_3) $\odot \bar{K}_n$ $m \geq 3, r \geq 3$. The vertex set and edge set of H_3 are defined as follows.

V (H_3) = {$v_{i,j}$, $v_{i,j,k}$ | 1 ≤ i ≤ 3, 1 ≤ j ≤ m, 1 ≤ k≤ n},

E(H_3) = {$v_{i,j}v_{i,,j}$+1 | 1 ≤ i ≤ 3, 1 ≤ j ≤ m − 1} ∪ {$v_{i,m}v_{i,i}$ | 1 ≤ i ≤ 3}, ∪ {$v_{i,j}v_{2,j}$, $v_{2,j}v_{3,j}$| 1 ≤ j ≤ m − 1} ∪{$v_{1,m}v_{2,m}$, $v_{2,m,j}v_{3m}$ } ∪ {$v_{i,j}$ $v_{i,j,k}$, $v_{i,m,k}$| 1 ≤ i ≤ 3, 1 ≤ j ≤ m − 1, 1 ≤ k ≤ n}.

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It is clear that H_3 has p = 3mn + 3m vertices and q = 3mn + 5m edges. Graph H_3 is given in Figure 1.

The following theorem gives the upper bound of the difference d in the super (a, d)EAMTL of H_3.

Theorem 2.1. Let H_3. =$(C_m \times P_3) \odot \bar{K}_n$ be the branched-prism graph on 3mn + 3m vertices and 3mn + 5m edges. If H_3 admits the super (a, d)-edge antimagic total labeling then d ≤ 2.

Proof. Suppose that H_3 has a super (a, d)-EAMTL defined by f : V (H_3)∪E(H_3) → {1, 2, · · · , 6mn + 8m}. The set of edge weight can be written as {a, a+d, · · · , a+ (q −1)d}, where q = 3mn+ 5m. The minimum possible edge-weight of this labeling is 1 + 2 + (3mn + 3m + 1) = 3mn + 3m + 4, and the maximum possible edge-weight is (3mn + 3m − 1) + (3mn + 3m) + ((3mn + 3m) + (3mn + 5m)) = 4(3mn + 3m) + 2m− 1 = 12mn + 14m − 1. Therefore,

$$a \geq 3mn + 3m + 4, \tag{1}$$

$$a + (q - 1)d = a + (3mn + 5m - 1)d \leq 12mn + 14m - 1. \tag{2}$$

From (1) and (2), we have the following inequality: 3mn + 3m + 4

+ (3mn + 5m − 1)d ≤ 12mn + 14m − 1,

$$d \leq \frac{(12mn + 14m - 1) - (3mn + 3m + 4)}{(3mn + 5m - 1)},$$

$$\leq \frac{9mn + 11m - 5}{3mn + 5m - 1}.$$

For m ≥ 3 and n ≥ 1, it is clear that d < 3. Therefore, if H_3 admits the super (a, d)-EAMTL then d ∈ {0, 1, 2}.

The following theorem gives the super (a, d)-EAMTL of H_3 for d = 1 and d = 2. The super (a, 0)-EAMTL ofH_2, or the super EMTL of H_2, has been obtained in [1]. *Azizu et al* etablished the super (a,d)-EAMTL of H_2 $for\ d = 1\ and\ d = 2$ in [2]..

Theorem 2.2. Let H_3 =$(C_m \times P_3)) \odot \bar{K}_n$ be the branched-prism graph on 3mn + 3m vertices and 3mn + 5m edges. For odd m, m ≥ 3 and n ≥ 1, there exist a super (a_1, 1)-edge antimagic total labeling and a super (a_2, 2)-edge antimagic total labeling of H_3, where a_1= 6mn + 6m +

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2 and a_2= 4mn + 3m + $\frac{(m+1)}{2}$ + 2.

Proof. Let $H_3 = (C_m \times P_3)) \odot \bar{K}_n$ be the branched-prism graph on 3mn + 3m vertices and 3mn + 5m edges, for odd m, m ≥ 3 and n ≥ 3. First, define the vertex labeling f : V (H_3) → { 1,2,….,3mn+3m} as follows.

$f(v_{1,i,j,}) = \frac{i+1}{2} + m(j-1)$, for odd i, $1 \leq i \leq m, 1 \leq j \leq n$,

$= \frac{m+i+1}{2} + m(j-1)$, for even i, 2 ≤ i ≤ m-1, 1 ≤ j ≤ n,

$f(v_{1,i}) = mn + \frac{i}{2}$ for even i, 2 ≤ i ≤ m − 1,

$= mn + \frac{m+i}{2}$, for odd i, 1 ≤ i ≤ m

$f(v_{2,,i})$ = mn+2m for i=1

$= mn + m + \frac{i-1}{2}$, for odd i, 3 ≤ i ≤ m,

$= mn + m + \frac{m+i-1}{2}$ for even i, 2 ≤ i ≤ m − 1.

$f(v_{3,i})$ = mn + 3m for i=2,

$= mn + 2m + \frac{m+i-2}{2}$ for odd i, 1 ≤ i ≤ m,

$= mn + 2m + \frac{i-2}{2}$ for even i, 4 ≤ i ≤ m-1,

$f(v_{2,i,j,})$ = mn + 3m +m + m(j − 1), for i=2, 1 ≤ j ≤ n

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$= mn + 3m + \frac{i-2}{2} + m(j-1)$, for even I, $4 \le i \le m-1, 1 \le j \le n$

$\frac{m+i-2}{2}$ $= mn+3m++ m(j-1)$, for odd i, $1 \le i \le m$, $1 \le j \le$

n $f(v_{3,,i,j,}) = 2mn + 4m + m(j-1)$ for $i=1$, $1 \le j \le n$

$= 2mn + 3m + \frac{m+i-1}{2} + m(j-1)$, for even i, $2 \le i \le m$, $1 \le j \le n$

$= 2mn + 3m + \frac{i-1}{2} + m(j-1)$, for odd i, $3 \le i \le m$, $1 \le j \le n$

Denote S = {f(x) + f(y) | xy ∈ E(H_3)} as the set of edge weights of the vertex labeling of H_3.

Therefore, S = { $mn + \frac{m+1}{2} + 1$, $mn + \frac{m+1}{2} + 2, \cdots,$ $4mn+5m+\frac{m+1}{2}$} Next, define the edge labeling f : E(H_3) → {3mn + 3m + 1, 3mn + 3m + 2, · · · , 7mn + 4m} as follows. The set of edge weights of the total labeling is denoted by W = {f(x)+f(y)+f(xy) | xy ∈ E(H_3)}. It can be seen that W = {s + f(xy) | xy ∈ E(H_3), s ∈ S}.

Consider the following cases.

Case 1. d = 2.

Define the minimum edge weight as

a = min{f(xy) | xy ∈ E(H_3)} + min{s | s ∈ S}

$= (3mn + 3m + 1) + (mn + 1 + \frac{m+1}{2})$

$= 4mn + 3m + \frac{m+1}{2} + 2.$

By choosing this minimum value of edge weight, we have the difference d = 2.

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Case 2. d = 1.

Let s ∈ S. Define the edge labeling of H_3as follows.

m+1 f(xy)

= s, for 2mn + 2m + 1 ≤ s ≤ 3mn + 3m +3 m +

2

= 4mn + s, for mn + 1 + $\frac{m+1}{2}$ ≤ s ≤ 3mn + 3m.

By defining this edge labeling, the minimum edge-weight is a

= s + f(xy) = (3mn + 3m + 1) + (3mn + 3m + 1) = 6mn + 6m + 2.

Therefore, there exist a super (a_1, 1)-EAMTL and a super (a_2, 2)-EAMTL of H_3, where a_1= 6mn + 6m + 2 and a_2= 4mn + 3m + $\frac{m+1}{2}$ + 2.. In Figure 2. we obtain a Super (100, 2)-EAMTL of $(C_m \times P_3))\odot \bar{K}_4$ . .

The black labels are for the vertices,while the blue ones are for the edges **3.**

Conclusion

This paper shows that the branched-prism graph H_3 =$(C_m \times P_3))\odot \bar{K}_n$ admits a super (6mn + 6m+2, 1)-EAMTL and (4mn+3m+ $\frac{m+1}{2}$ +2, 2)-EAMTL for odd m, m ≥ 3, n ≥ 1. Combining this result with [1] and [2] we have the super (a, d)-EAMTL of the branched-prism graph H_3 $(slitly\ differ\ from\ Figer$ $H_2)$ d ∈ {0, 1,2} .

References

[1] K.A. Azizu, L. Yulianti, S. Sy, N. Narwen, On super edge magic total labeling of the branched-prism graph, International Journal of Progressive Sciences and Technologies 16
.

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10.48047/jocaaa.2022.30.02.47

(2) (2019), 203 – 206.

[2] K.A. Azizu, L. Yulianti, S. Sy, N. Narwen, On super (a,d) - edge antimagic total labeling of the branched-prism graph. $(C_m \times P_2) \odot \bar{K}_n$, Indonesian Journal of Combinatorics 5(1) (2021), 11 – 16.

[3] M. Baca, M. Miller, Super Edge-Antimagic Graphs: A wealth of Problems and some solutions. Brown Walker Press (2008).

[4] H. Enomoto, A.S. Llado,T. Nakamigaw and G.Ringel, Super magic graphs, SUT, J. Math., 2(1998)105-109

[5] J. A. Gallian, A Dynamic survey of graph labeling. Electron. J. Combin. (2020), #DS6.

[6] N. Hartsfield, G. Ringel, Pearls in Graph Theory, Academic Press. Boston (1990).

[7] A.Kotzig . and A.Rosa ,Pearls in graph theory Acadimic Press, New Yark, 2nd adition (2001).

[8] A. Kotzig and A. Rosa , Magic valuations of finite graphs ,Canad. Math. Bull. 13 (1970)451-461

[9] A.Kotzig . and A.Rosa, Magic valuations of complete graphs,Publications due centre, de Recherches Mathematiques Universite de Montreal 175(1972)

[10]G. Ringel, Lebeling Problems, Combinatorics and graph theory and Algorithm, Vol. I.D.(Kalamazoo M.I. 1999)723-728, New Issues Press Kalamazoo M.I. (1999)

[11] R. Simanjuntak, M. Miller, F. Bertault, Two new (a, d)-antimagic graph labeling. Proc. of the Eleventh Australasian Workshop on Combinatorial Algorithms, (2000), 179 – 189.

[12] K.A. Sugeng, Super edge antimagic total labeling, Util. Math. 71 (2006), 131 – 141.

[13] K. Wijaya and E.T. Baskoro, Edge magic labeling of a prodect of two graphs, Proc. Seminar MIPA, ITB Bandung Oct.(2000)

[14] D.B. West,An introduction to graph theory,Prentice-Hall (1996)