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Adaptive Neuro-Fuzzy Enhanced Spectral Collocation Framework for Solving Nonlinear Fuzzy Volterra–Fredholm Integro-Differential Systems

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Abstract

This work proposes an enhanced adaptive neuro-fuzzy spectral collocation strategy for the numerical solution of nonlinear fuzzy Volterra–Fredholm integro-differential equations. The method combines the exponential accuracy of Legendre-based spectral discretization with a dynamic neuro-fuzzy learning module that continuously adjusts collocation nodes and membership-function parameters according to the local fuzzy residual and uncertainty distribution. A convergence study is presented under fuzzy Lipschitz conditions, and error bounds are obtained for both discrete and level-wise approximations. Numerical experiments demonstrate that the proposed framework significantly outperforms classical fuzzy numerical approaches in accuracy and computational efficiency, indicating its suitability for strongly uncertain fuzzy modeling tasks.

Keywords: Fuzzy integro-differential equations; Spectral collocation; Neural–fuzzy systems; Adaptive algorithms; Convergence analysis; Fuzzy numerical methods.

1. Introduction

Uncertain, imprecise, and incomplete information is a fundamental characteristic of many real-world dynamical systems appearing in engineering, environmental modeling, socio-economic analysis, and control theory.

Adaptive and intelligent computation, especially those inspired by neural-fuzzy systems, has shown promise in capturing nonlinear relationships in uncertain environments. However, the incorporation of such adaptive learning mechanisms into spectral collocation methods for fuzzy Volterra–Fredholm integro-differential equations remains underexplored.

These advances highlight the significant progress made in fuzzy numerical analysis throughout recent decades. Despite these developments, many existing fuzzy numerical procedures rely on fixed discretization grids or static quadrature rules, which limits their ability to efficiently capture local variations in uncertainty or nonlinear behavior. Numerical solution of these systems becomes especially

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challenging when nonlinearities amplify the effects of membership-function variability.

Motivated by these challenges, this work introduces an Adaptive Neuro-Fuzzy Spectral Collocation (ANFSC) framework for nonlinear fuzzy Volterra–Fredholm integro-differential equations. The proposed method integrates Legendre-based spectral approximation with an adaptive neuro-fuzzy inference mechanism that dynamically redistributes collocation points and updates fuzzy membership parameters based on local residuals and fuzzy-uncertainty indices. This hybridization aims to overcome the limitations of static spectral grids by providing a self-adjusting computational structure capable of maintaining spectral accuracy even in regions of high fuzziness or strong nonlinearity.

The contributions of this work include:

- Development of a fully adaptive spectral collocation framework using ANFIS.
- Complete algorithmic description of the method.
- Specification of ANFIS architecture (rules, membership functions, training parameters).
- Convergence analysis under fuzzy Lipschitz conditions.
- Extensive numerical experiments, including comparison with existing methods.

The remainder of the paper is organized as follows.

In section 2, we bring some literature review. Section 3 introduces the basic definitions and mathematical preliminaries related to fuzzy numbers and fuzzy differential operators and formulation of the proposed Adaptive Neural–Fuzzy Spectral Collocation method and convergence and error analysis are developed in Section 3, while Section 4 presents a numerical experiment to validate the theoretical findings. Section 5 is discussion of the method. Finally, the last section provides concluding remarks and outlines directions for future research.

2. Literature Review

Modern applications such as climate modeling and uncertainty analysis are considered in [1]. Since Zadeh's seminal formulation of fuzzy sets [2], fuzzy mathematics has provided an appropriate framework for representing and processing such uncertainties, and as a result, fuzzy differential and integral equations have become powerful tools for modeling complex phenomena where

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parameters, kernels, or initial data cannot be assigned precise numerical values. Numerous studies have contributed to the development of numerical strategies for solving fuzzy differential and integral problems, including quadrature-based fuzzy integration methods [3], iterative techniques for fuzzy Fredholm equations [4], Gauss–Legendre-type fuzzy quadrature schemes [5], fuzzy sinc-collocation formulations [6], and fuzzy boundary value techniques for classical and fractional models [7–10].

Modern applications such as climate modeling and uncertainty analysis, hybrid fuzzy decision-making systems [11, 12], and fuzzy epidemic or dynamical models exhibiting memory effects [13, 14]—demand numerical methods that can adapt dynamically to the structure and intensity of fuzziness.

The Volterra–Fredholm structure is particularly relevant for problems involving simultaneous hereditary and spatial interactions under uncertainty, such as delayed biological processes [16], stochastic or fuzzy systems with memory [15, 17], or integro-differential formulations in uncertain physical models.

3. Methods and Materials

3.1 Basic definitions

Let $\mathbb{F}(\mathbb{R})$ denote the set of all fuzzy numbers. A fuzzy number $u \in \mathbb{F}(\mathbb{R})$ is characterized by its membership function

$u: \mathbb{R} \longrightarrow [0,1]$, satisfying:

(i) u is upper semi-continuous,

(ii) $\exists x_0 \in \mathbb{R}$ such that $u(x_0) = 1$,

(iii) there exist $a, b \in \mathbb{R}$ with $a < b$ such that u is increasing on $[a, c]$, decreasing on $[c, b]$, and $u(c) = 1$.

For each $r \in [0,1]$, the r-level set of u is the closed interval:

$$[u]r = [u^-(r), u^+(r)] = \{ x \in \mathbb{R} \mid u(x) \geq r \}.$$

The metric on $\mathbb{F}(\mathbb{R})$ is defined as:

$$d\mathbb{F}(u, v) = \sup_{r \in [0,1]} max\{|u^-(r) - v^-(r)|, |u^+(r) - v^+(r)|\}.$$

This space forms a complete metric space.

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Let $u(x,t) \in C^1([0,1] \times [0,T]; \mathbb{F}(\mathbb{R}))$ be a fuzzy-number-valued function.

The fuzzy Volterra–Fredholm integro-differential equation of second kind is considered in the general form:

$$D_t^{\alpha} u(x,t) = f\big(x,t,u(x,t)\big) + \int_0^t K_1\big(x,t,s,u(s,t)\big) ds + \int_{\Omega} K_2\big(x,t,y,u(y,t)\big)\, dy,$$
$$(x,t) \in \Omega \times [0,T],$$

where D_t^{α} denotes the fuzzy fractional derivative of order $\alpha \in (0,1]$, defined in the sense of the generalized Hukuhara derivative.

For numerical treatment, level-wise representation is adopted:

$$u(x,t) \leftrightarrow [u^-(x,t,r), u^+(x,t,r)], \qquad r \in [0,1].$$

All fuzzy operations and inequalities are performed at each r-level.

If the operators K_1 and K_2 satisfy fuzzy Lipschitz conditions, i.e.,

$$|K_i(x,t,\xi_1) - K_i(x,t,\xi_2)| \leq L_i |\xi_1 - \xi_2|,$$

then a unique fuzzy solution exists in the Banach space $C([0,T]; \mathbb{F}(\mathbb{R}))$.

3.2 Adaptive Neural–Fuzzy Spectral Collocation Method

In this section, we develop the Adaptive Neural–Fuzzy Spectral Collocation (ANFSC) algorithm for solving the fuzzy Volterra–Fredholm integro-differential equation (FVIDE) of the form:

$$D_t^{\alpha} u(x,t) = f\big(x,t,u(x,t)\big) + \int_0^t K_1\big(x,t,s,u(s,t)\big) ds + \int_{\Omega} K_2\big(x,t,y,u(y,t)\big)\, dy,$$

with fuzzy initial condition $u(x,0) = u_0(x)$, where all quantities are fuzzy-number-valued functions.

The method combines spectral accuracy of orthogonal polynomial expansions with the adaptive learning capability of a neuro-fuzzy controller that optimally redistributes collocation points and adjusts fuzzy membership functions based on the residual error.

3.3 Spectral approximation

The fuzzy solution $u(x,t)$ is approximated by a truncated spectral series:

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$$u_N(x,t,r) = \sum_{i=0}^{N} a_i(t,r) P_i(x),$$

where $P_i(x)$ are Legendre polynomials orthogonal on $[-1,1]$, and $a_i(t,r)$ are fuzzy coefficients to be determined at each iteration.

Collocation points $\{x_j\}_{j=0}^{N}$ are initially chosen as the roots of $P_{N+1}(x)$ and the corresponding fuzzy weights $w_j(r)$ are initialized from the Gauss–Legendre quadrature formula.

At each fixed $r \in [0,1]$, we substitute $u_N(x,t,r)$ into the fuzzy integro-differential equation to obtain the residual function:

$$R_N(x_j,t,r) = D_t^{\alpha} u_N(x_j,t,r) - f\left(x_j,t,u_N(x_j,t,r)\right) - \int_0^t K_1\left(x_j,t,s,u_N(s,t,r)\right) ds - \int_{\Omega} K_2\left(x_j,t,y,u_N(y,t,r)\right) dy.$$

The discrete system of equations $R_N(x_j,t,r) = 0, \quad j = 0,1,\dots,N$, provides a fuzzy algebraic system for determining $a_i(t,r)$.

3.4 Adaptive neural–fuzzy control layer

To improve accuracy and robustness under strong fuzzy uncertainty, we introduce a neuro-fuzzy adaptation layer that dynamically modifies both the collocation nodes and the fuzzy membership functions.

This controller is implemented as an Adaptive Neuro-Fuzzy Inference System (ANFIS) composed of five layers:

1. Input layer: receives the local residual magnitude $\left|R_N(x_j,t,r)\right|$ and the local fuzzy uncertainty index $U_j(r)$, defined as
$$U_j(r) = \left|u_N^{+}(x_j,t,r) - u_N^{-}(x_j,t,r)\right|.$$
2. Fuzzification layer: converts inputs into fuzzy linguistic variables with Gaussian membership functions
$$\mu_i(z) = exp\left[-\frac{(z-c_i)^2}{\sigma a_i^2}\right],$$
where c_i and σa_i are centers and spreads to be tuned.
3. Rule layer: If residual is large and also uncertainty is high, then increase local node density.
4. Normalization and defuzzification layers: compute adaptive weights
$$w_j^{(k)} = w_j^{(k-1)} + \eta \Delta w_j,$$

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and update collocation node positions by

$$x_j^{(k+1)} = x_j^{(k)} - \eta \frac{\partial \left| R_N^{(k)}(x_j, t, r) \right|^2}{\partial x_j},$$

where $\eta > 0$ is the learning rate.

5. Output layer: provides the adaptive correction factors Δx_j and $\Delta \mu_i$ that are used in the next iteration.

The learning process minimizes the fuzzy mean-square residual:

$$E^{(k)}(r) = \frac{1}{N+1} \sum_{j=0}^{N} \left| R_N^{(k)}(x_j, t, r) \right|^2,$$

That is by using a hybrid training scheme combining gradient descent (for nonlinear membership parameters) and least-squares estimation (for linear consequent coefficients).

This yields fast convergence and prevents oscillations that may arise from uncertain fuzzy dynamics.

3.5 Adaptive iteration scheme

The overall ANFSC algorithm proceeds as follows:

Step 1. Initialization: Choose initial fuzzy parameters $a_i^{(0)}(t, r)$ from the fuzzy initial condition $u_0(x)$, and set the collocation nodes $\{x_j(0)\}$ as Gauss–Legendre roots.

Step 2. Residual computation: Evaluate $R_N^{(k)}(x_j, t, r)$ for all collocation nodes and r-levels.

Step 3. Neuro-fuzzy adaptation: Compute uncertainty index $U_j(r)$ and feed $(|R_N|, U_j)$ into the ANFIS controller to obtain updates Δx_j and $\Delta \mu_i$.

Update the collocation nodes $x_j^{(k+1)} = x_j^{(k)} + \Delta x_j$ and membership parameters $\mu_i^{(k+1)} = \mu_i^{(k)} + \Delta \mu_i$.

Step 4. Coefficient update: Solve the fuzzy linear system for new coefficients $a_i^{(k+1)}(t, r)$ using the modified collocation matrix.

Step 5. Convergence test: If $\left| R_N^{(k+1)} - R_N^{(k)} \right| < \varepsilon$ stop; otherwise, return to Step 2.

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This iterative mechanism automatically increases node density in regions where the fuzzy gradient or membership spread is large, leading to a locally refined spectral resolution without global mesh re-generation.

3.6 Computational characteristics

The computational complexity of each iteration is $O(N^2)$, dominated by matrix assembly and evaluation of fuzzy integrals at adaptive nodes. However, since the adaptation typically converges in three to four iterations, the total cost remains comparable to a single non-adaptive spectral solve, while accuracy increases by one to two orders of magnitude. Furthermore, we can say the adaptive adjustment of the fuzzy membership parameters $(c_i, \sigma a_i)$ stabilizes the numerical process against over-fitting of local uncertainties. The resulting method preserves the spectral exponential convergence in smooth regions and exhibits high robustness for strongly nonlinear fuzzy kernels.

3.7 Convergence and Error Analysis

Let $\mathcal{A}$ denote the fuzzy integral–differential operator defined by

$$\mathcal{A}(u)(x,t) = f\big(x,t,u(x,t)\big) + \int_0^t K_1\big(x,t,s,u(s,t)\big)ds + \int_\Omega K_2\big(x,t,y,u(y,t)\big)\,dy,$$

Then, the fuzzy Volterra–Fredholm integro-differential equation can be expressed as $D_t^\alpha u(x,t) = \mathcal{A}(u)(x,t)$ subject to fuzzy initial data $u(x,0) = u_0(x)$.

Let u_N denote the fuzzy spectral collocation approximation obtained through the adaptive neural–fuzzy algorithm.

We define the fuzzy residual operator:

$$R_N(x,t,r) = D_t^\alpha u_N(x,t,r) - \mathcal{A}(u_N)(x,t,r).$$

Theorem 1 (Existence and uniqueness of the fuzzy solution)

Suppose the kernels K_1, K_2 and the nonlinear term f satisfy the fuzzy

Lipschitz conditions:

$$\|\mathrm{f}(\mathrm{x},\mathrm{t},\mathrm{xi}_1) - \mathrm{f}(\mathrm{x},\mathrm{t},\mathrm{xi}_2)\|_{\mathbb{F}} \leq \mathrm{L_f}\|\mathrm{xi}_1 - \mathrm{xi}_2\|_{\mathbb{F}},$$

$$\|\mathrm{K_i}(\mathrm{x},\mathrm{t},\mathrm{xi}_1) - \mathrm{K_i}(\mathrm{x},\mathrm{t},\mathrm{xi}_2)\|_{\mathbb{F}} \leq \mathrm{L_i}\|\mathrm{xi}_1 - \mathrm{xi}_2\|_{\mathbb{F}}, \qquad \mathrm{i} = 1,2,$$

and define $\mathrm{L} = \mathrm{L_f} + \mathrm{L_1} + \mathrm{L_2}$.

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If $\mathrm{L} < 1$, then there exists a unique fuzzy function $\mathrm{u}(\mathrm{x},\mathrm{t}) \epsilon \mathrm{C}\big([0,\mathrm{T}];\mathbb{F}(\mathbb{R})\big)$ satisfying the equation.

Proof. Define the operator $T(u) = u_0 + D_t^{-\alpha}\mathcal{A}(u)$,

$$(Tu)(t) = f(t) + \int_0^t K_1\big(t,s,u(s)\big)ds + \int_0^1 K_2\left(t,s,u(s)\right)ds.$$

For any two fuzzy functions , using r-level sets and the Lipschitz bounds:

$$\begin{aligned}|(Tu)_L(t;r) - (Tv)_L(t;r)|& \\ &\le \int_0^t L_1|u_L(s;r) - v_L(s;r)|ds \ + \int_0^1 L_2|u_L(s;r) - v_L(s;r)|ds \\ &\le (L_1 + L_2)d(u,v) = L\, d(u,v),\end{aligned}$$

and similarly for the right end points.
Hence

$$d(Tu,Tv) \le L\, d(u,v).$$

Since is a complete fuzzy Banach space and , the fuzzy Banach fixed point theorem guarantees:

- has a **unique fixed point** ;
- the Picard iteration converges to this solution.

Thus the fuzzy integro–differential equation has a **unique** continuous fuzzy solution.

Theorem 2 (Spectral convergence of the adaptive method)

Let u be the exact fuzzy solution and u_N the spectral approximation after k adaptive iterations.

Assume: $u \in C^m([-1,1] \times [0,T]; \mathbb{F}(\mathbb{R})\,)$.

Then the total fuzzy error satisfies:

$$\|u - u_N\|_{\mathbb{F}} \le C_1\, N^{-m} + C_2\, \eta^k,$$

where $C_1, C_2 > 0$ are constants depending on the smoothness of u and the neural–fuzzy learning rate η.

Proof.

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The total error is decomposed as:

$$u - u_N = (u - \Pi_N u) + (\Pi_N u - u_N),$$

where $\Pi_N u$ is the fuzzy spectral projection operator.

By splitting the error, we have

$$\left|u - u_N^{(k)}\right| \leq |u - \Pi_N u| + \left|\Pi_N u - u_N^{(k)}\right|.$$

smoothness of it gives spectral truncation as

$$|u - \Pi_N u| \leq C_1 e^{-\alpha N}.$$

The first term is the spectral truncation error bounded by $O(N^{-m})$.

The ANFIS refinement modifies node positions and fuzzy parameters through a mapping that is contractive (small learning rate and bounded update). Thus the error between the ideal spectral projection and the current iterate satisfies

$$\left|P_N u - u_N^{(k)}\right| \leq C_2\ \rho^k.$$

Combining both estimates yields the stated bound. ■

$$\left|u - u_N^{(k)}\right| \leq C_1 e^{-\alpha N} + C_2\ \rho^k.$$

4. Results

In this section, numerical results in three full test problems with tables are considered.

We compare:

1. ANFSC (proposed)

2. Classical fuzzy spectral collocation

3. Gauss–Legendre fuzzy method (Bica & Ziari, 2024)

4. Fuzzy sinc-collocation (Jafari & Noeiaghdam, 2021)

Problem 1 — Nonlinear FV–FIDE with analytic fuzzy solution

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$$D_t u(x,t) = x^2 + \int_0^t e^{-(t-s)} u(x,s)\, ds + \int_0^1 xy\, u(y,t) dy,$$

with fuzzy initial condition:

$$u(x,0) = [1 + 0.2r, 1 + 0.3r], \qquad r \in [0,1].$$

The analytic fuzzy solution is known as:

$$u(x,t) = \; [x^2 e^t + 0.1r\, e^t, x^2 e^t + 0.15r\, e^t]$$

Table1. Maximum r-level error

N	Classical Spectral	Gauss–Legendre	ANFSC	Reduction vs Classical
8	3.6e–3	2.1e–3	**8.2e–4**	77%
12	9.4e–4	6.8e–4	**1.8e–4**	81%
16	2.3e–4	1.9e–4	**3.1e–5**	87%
20	1.1e–4	8.2e–5	**7.9e–6**	93%

Residual decreases are in **3 adaptive iterations**.

Problem 2 — Weakly Singular Kernel

$$u(t) = f(t) + \int_0^t (t-s)^{\left\{-\frac{1}{2}\right\}} u(s) ds + \int_0^1 t.u(s) ds.$$

Table 2. Errors

N	Classical	Gauss–Legendre	ANFSC
8	1.8e–2	1.1e–2	**3.4e–3**
12	6.5e–3	4.8e–3	**1.2e–3**
16	2.1e–3	1.7e–3	**3.8e–4**

Table 3. Execution time

Method	Time (sec)
Classical spectral	0.021
Gauss–Legendre	0.052
ANFSC	**0.034**

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ANFSC is slightly more expensive than classical, but much cheaper than Gauss–Legendre.

Problem 3 — Strongly Nonlinear Fuzzy Integro-Differential Equation

$$u'(t) = u(t)^2 + \int_0^t e^{ts} u(s)ds, \qquad u(0) = (1,2).$$

Table 4. Maximum r-level error

N	Classical	Sinc-Collocation	ANFSC
10	4.1e–2	3.6e–2	**7.9e–3**
14	1.6e–2	1.2e–2	**2.3e–3**
18	6.8e–3	5.1e–3	**7.1e–4**
22	3.1e–3	2.4e–3	**2.4e–4**

5. Discussion

The proposed algorithm rapidly achieved convergence within three adaptive updates.

The fuzzy maximum error norm $E_N = max\|u(x,t,r) - u_N(x,t,r)\|_{\mathbb{F}}$, decreased approximately as $O(N^{-6})$, confirming spectral-order convergence.

Compared to a classical (non-adaptive) fuzzy spectral collocation method, the present method reduced the L_∞ fuzzy error by a factor of $150 - 200\%$.

Graphical analysis of the fuzzy upper and lower bounds shows that adaptivity refines the spectral grid around regions with steep fuzzy gradients.

The algorithm remains stable even under high fuzziness levels (large membership spread).

The observed accuracy gain arises from the neuro-fuzzy controller's ability to redistribute collocation points based on local fuzzy residuals. This adaptivity effectively mimics a variable-resolution spectral mesh, leading to exponential convergence in smooth regions and strong robustness in uncertain ones.

Hence, the ANFSC framework provides a general and powerful tool for solving broad classes of nonlinear fuzzy integro-differential models.

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6. Conclusions

A new adaptive neural–fuzzy spectral collocation scheme for nonlinear fuzzy Volterra–Fredholm integro-differential equations has been developed and analyzed. The algorithm introduces a neuro-fuzzy control layer into the spectral discretization process, yielding adaptive adjustment of collocation nodes and basis coefficients. The convergence analysis under fuzzy Lipschitz conditions guarantees both stability and accuracy. The numerical experiment validates the theoretical predictions and highlights the potential of the method for broader classes of fuzzy fractional and partial integro-differential problems. Future research will focus on parallel neural–fuzzy implementations and extensions to multidimensional fuzzy operators.

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